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Reg. No. : ....

Name : ....

Fourth Semester B.Tech. Degree Examination, September 2020

13.401 : PROBABILITY, RANDOM PROCESSES AND NUMERICAL TECHNIQUES (FR)

(2013 scheme)

Time : 3 Hours Max. Marks : 100

PART – A

Answer **all** questions. Each question carries **4** marks :

1. Find the value of K if $f(x)=k(2-x),\ 0<x<2$ is a probability density function of a random variable.

2. A random variable has uniform distribution over $(-3,3)$. Compute

   (a) $P[X<2]$

   (b) $P[|X|<2]$.

3. If $X(t)$ is a WSS process with $E(X(t))=2$ and $R(\tau)=4+e^{-\frac{|\tau|}{10}}$ find the mean of $S=\int_0^1 X(t)dx$.

4. The autocorrelation function of a stationary process $\{(X(t))\}$ is given by $R(\tau)=\frac{25\tau^2+36}{6.25\tau^2+4}$ find mean and variance of the process $\{(X(t))\}$.

5. Solve by Gauss elimination method $2x+3y-z=5,\ 4x+4y-3z=3, 2x-3y+2z=2$.

P.T.O.

PART – B

Answer **one full** question from each module. Each question carries **20** marks :

Module — I

6. (a) If $f(x)=\begin{cases} x & 0<x<1 \\ 2-x & 1\leq x<2 \end{cases}$ is the probability density function of a random variable. Find mean and distribution function. **7**

(b) If 5% of the electric bulbs manufactured by a company are defective. Use Poisson Distribution to find the probability that in a sample of 100 bulbs

(i) None is defective

(ii) 5 bulbs are defective. **7**

(c) In an examination 30% of the candidates obtained marks below 40 and 10% of the candidates got above 75 marks. Assuming that the marks are normally distributed . find the mean and standard deviation of the distribution. **6**

OR

7. (a) A Random variable has the following probability distribution

X	0	1	2	3
$f(x)$	k/2	k/3	$\frac{k+1}{3}$	$\frac{2k+1}{3}$

Find (i) k (ii) Mean (iii) variance (iv) Distribution function. **7**

(b) The amount of time that a surveillance camera will run without having to be retest is a random variable having the exponential distribution with mean 120 days. Find the probability that such a camera will

(i) Have to be retested in less than 24 days

(ii) Not have to be retested at least 120 days. **7**

(c) A box contains 100 cell phones, 20 of which are defective. 10 cell phones are selected for inspection. Find the probability that

(i) at least one is defective

(ii) at most three are defective

(iii) none of them are defective

(iv) all of them are defective. **6**

Module — II

8. (a) If $f(x, y)=Ke^{-(2x+y)}\ x\geq 0, y\geq 0$ is a joint probability density function of two dimensional random variable. Find

   (i) K (ii) Conditional Distributions of X and Y. **7**

   (b) The joint probability density function of X and Y is given by

   $f(x)=(1-x)(1-y)$ for $0<x<1, 0<y<1$. Find $P\left(0<x<\frac{1}{2},\frac{1}{2}<y<1\right)$ prove that X and Y are independent. **7**

   (c) Calculate the coefficient of correlation for the following data. **6**

X	22	26	29	30	31	31	34	35
Y	20	20	21	29	27	24	27	31

OR

9. (a) If $X(t)=\sin(\omega t+Y)$ where ω is a constant and Y is uniformly distributed on $(0,2\pi)$. Prove that $X(t)$ is WSS. **10**

   (b) If $X(t)$ and $Y(t)$ are independent zero mean WSS process and $Z(t)=X(t)+Y(t)$, Show that $Z(t)$ is WSS. **10**

Module — III

10. (a) If the Power Spector density of a WSS process is a $S(\omega)=\begin{vmatrix}\frac{b}{a}(a-|\omega|) & |\omega|\leq a \\ 0 & |\omega|>a\end{vmatrix}$ where a and b are constants, find the auto correlation function of the process. **10**

   (b) Find the average power of the random process $\{(X(t))\}$ if the Power Spectral Density is given by $S(w)=\frac{10w^2+35}{(w^2+4)(w^2+9)}$. **10**

OR

11. (a) If the auto correlation function of a WSS process is $R(\tau)=\rho e^{-\rho|\tau| e>0}$ Find the Powerspectral density. **10**

(b) If the customers arrive at a counter in accordance with Poisson Process with mean rate of 2 per minutes. Find the probability that the interval between 2 consecutive arrivals is

(i) more than 1 minutes

(ii) between 1 and 2 minutes

(iii) 4 minutes or less. **10**

Module — IV

12. (a) Using Newton Raphson method to solve the equation $x^3+x-1=0$ correct to 4 decimalplaces. **7**

(b) Using Lagrange's interpolation formula find the value of y when $x=10$ for the following table **7**

X	5	6	9	11
Y	12	13	14	16

(c) Evaluate $\int_0^6 \frac{1}{1+x^2} dx$ using

(i) Trapezoidal rule (ii) Simpson's rule with 6 equal intervals. **6**

OR

13. (a) Solve by Gauss Siedel Iteration Method

$28x+4y-z=32, 2x+17y+4z=35, x+3y+10z=24$. **7**

(b) Construct the Newton's forward interpolation polynomial which takes the following values **7**

X	0	1	2	3
Y	1	2	1	10

(c) Find the root of $x^3-x-11=0$ that lies between 2 and 3 correct to two decimal places by bisection method. **6**