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Reg. No. : ....

Name : ....

Fifth Semester B.Tech. Degree Examination, December 2018

(2013 Scheme)

13.501 : ENGINEERING MATHEMATICS – IV (AFRT)

(Complex Analysis and Linear Algebra)

Time : 3 Hours Max. Marks : 100

PART – A

Answer **all** questions. **Each** question carries **4** marks.

1. Define analytic function. State the necessary condition for a function f(z) to be analytic at a point. Use this to check whether f(z) = sinhz is analytic.

2. Find the image of $x<c$ $(c>0)$ under the transformation $w = 1/z$. What is the image of $c = 0$?

3. Show that $\int_C \frac{dz}{(z-a)^n} = \begin{cases} 2\pi i & \text{if } n=1 \\ 0 & \text{otherwise} \end{cases}$ where C is the circle $|z-a| = r$.

4. Let $A = \begin{bmatrix} -7 & -2 & -9 \\ 6 & 4 & 8 \\ 4 & 5 & 4 \end{bmatrix}$ and $W = \begin{bmatrix} -2 \\ 5 \\ -2 \end{bmatrix}$. Determine if W is in C(A) and N(A).

5. Show that B = { (2, 1, – 1), (–1, 1, –1), (0, 3, 3) } forms an orthogonal basis for R^3. Also find the co-ordinate vector of (–7, 1, –9) relative to the basis B.

P.T.O.

PART – B

Answer **any one full** question from **each** Module.. **Each** question carries **20** marks.

Module – I

6. a) If f (z) is analytic prove that $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$.

   b) Show that $u = e^{2x}(x\cos 2y - y\sin 2y)$ is a harmonic function. Find the analytic function f(z)= u + iv satisfying f(1) = 3i.

   c) Under the transformation $w = \frac{z-i}{1-iz}$, what is the image of | z | = 1.

7. a) Show that $f(z) = \frac{xy}{x^2+y^2}$, $z \neq 0$ and f(0) = 0 satisfy CR equations, but not differentiable at z = 0.

   b) If $u+v = \frac{\sin 2x}{\cosh 2y - \cos 2x}$ find f(z) = u + iv which is analytic.

   c) Find the bilinear transformation which maps (1, i, – 1) into the point (i, – 1, – i).

Module – II

8. a) Expand $f(z) = \frac{z^2-1}{z^2+5z+6}$ in the region 2 < | z | < 3.

   b) Use Cauchy's integral formula to evaluate $\oint_{|z|=3} \frac{5z^2+z-2}{(z-2)^3}\,dz$.

   c) Prove that $\int_0^{2\pi} \frac{\cos 3\theta\, d\theta}{5-4\cos\theta} = \frac{\pi}{12}$.

9. a) Find the poles and residues of $\frac{z^2-2z}{(z+1)^2(z^2+1)}$.

b) Evaluate $\oint_C \frac{e^{2z}}{(z-1)^3(z+3)}dz$ where C is $|z-1| = 2$.

c) Evaluate $\int_0^\infty \frac{\cos x dx}{(x^2+a^2)^2}$.

Module – III

10. a) Find the dimensions and a spanning set of the null space and the column space of $A = \begin{bmatrix} -3 & 6 & -1 & -7 \\ -1 & 2 & -3 & 4 \\ 1 & -2 & 3 & -4 \\ 1 & -2 & 9 & -1 \end{bmatrix}$

b) Show that $T : R^3 \to R^3$ defined by $T(x, y, z) = (x + z, x + y + 2z, 2x + y + 3z)$ is a linear transformation. Find a basis for the Kernel of T and Range of T.

c) Let $V = R^4$ and W be a subspace generated by (1, 4, –1, 3), (2, 1, – 3, 1) and (0, 2, 1, – 5).

i) Find a basis and dim(W).

ii) Extend the basis of W to a basis of R^4.

11. a) Find a matrix A such that W = C (A), where $W = \{(2a + 3b, a + b - 2c, 4a + b, 2a - b - c) : a, b, c \in R\}$

b) Check whether that the functions $f(t) = t^3 - 6t^2 - 2t + 5$, $g(t) = t^3 + 4t^2 - 3t + 4$ and $h(t) = 2t^3 - 7t^2 - 7t + 9$ are linearly dependent.

c) Suppose a linear transformation maps (1, 1) to (2, 2) and (2, 0) to (0, 0). Find T(v) when

i) v = (2, 1)

ii) v = (3, – 1)

iii) v = (– 1, 1) and

iv) v = (a, b).

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Module – IV

12. a) Find an orthonormal basis for the subspace spanned by (1, 1, 1, 1) (1, 2, 4, 5) and (1, – 3, – 4, – 2) in R^4.

b) Find the least square solution of AX = b for $A = \begin{bmatrix} 2 & 3 \\ -1 & 3 \\ 2 & 4 \end{bmatrix}$ $b = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$. Also find the least square error.

c) Find a maxima or minima of $x^2 + 2y^2 + 3z^2 + 2xy + 2yz - 2xy$ subject to the constraint $X^TX = 1$ where $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$.

13. a) Find the singular value decomposition of $A = \begin{bmatrix} 1 & -1 \\ -2 & 2 \\ 2 & -2 \end{bmatrix}$.

b) Find the angle between the functions $f(t) = t^2 + t + 2$ and $g(t) = 3t - 2$ in the polynomial space P (t) with innerproduct $\langle f, g \rangle = \int_0^1 f(t)\, g(t)\, dt$.

c) Write y as a sum of a vector in W = span $\{u_1, u_2\}$ and a vector in W^T where $y = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$, $u_1 = \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix}$, $u_2 = \begin{bmatrix} 5 \\ 1 \\ 4 \end{bmatrix}$.

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