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G – 3586

Reg. No. : ....

Name : ....

Fourth Semester B.Tech. Degree Examination, June 2019

(2013 Scheme)

13.401 : PROBABILITY, RANDOM PROCESSES AND NUMERICAL TECHNIQUES (FR)

Time : 3 Hours

Max. Marks : 100

PART – A

Answer **all** questions. Each question carries **4** marks. :

1. If $f(x)=\dfrac{k}{2^x}$ is a probability distribution of a random variable which can take values $x=0,\ 1, 2, 3, 4$. Find K and Mean of the distribution.

2. Find the mean and variance of the probability distribution with density function $f(x)=Ke^{-\frac{1}{8}(x^2+8x+16)}$.

3. The customers arrive at a bank according to a Poisson Process with mean rate of 2 minutes. Find the probability that during an 1 minute interval no customers arrive.

4. The autocorrelation function of a stationary process $\{(X(t))\}$ is given by $R(\tau)=2+4e^{-2|\tau|}$. Find mean and variance of the process $\{(X(t))\}$.

P.T.O.

5. Using Lagrange's interpolation formula find the value of y when $x = 9$ for the following data

X	5	6	11
Y	12	13	16

(5 × 4 = 20 Marks)

PART – B

Answer **one full** question from **each** Module. Each question carries **20** marks.

Module – I

6. (a) If $f(x) = \begin{cases} 0 & x < 2 \\ \frac{1}{18}(2x+3) & 2 \le x < 4 \\ 0 & x > 4 \end{cases}$ is the probability density function of a random variable. Find Mean and distribution function. **7**

(b) Human errors is given as the reason for 75% of all accidents in a plant. Use Binomial distribution to find the probability that human error will be given as the reason for 2 of the next 4 accidents. **7**

(c) The mean weight of 500 students at a certain school is 50 kg and the standard deviation is 6 kg. Assuming that the weights are normally distributed, find the expected number of students weighing

(i) between 40 and 50 kg

(ii) more than 60 kg. **6**

OR

7. (a) A Random variable has the following probability distribution **7**

X	-2	-1	0	1	2	3
$f(x)$	$1/10$	k	$1/5$	$2k$	$3/10$	$3k$

Find :

(i) k

(ii) Mean

(iii) Variance

(iv) $P(-2 < X < 2)$.

(b) The number of cell phones sold daily in a shop is uniformly distributed with a minimum of 50 phones and a maximum of 100 phones. Find the probability that :

(i) the daily sales will fall between 70 and 80 phones

(ii) at least 75 phones are sold on a given day

(iii) at most 70 phones are sold on a given day. **7**

(c) The monthly breakdown of a computer follows Poisson distribution with mean 1.2. Find the probability that this computer will function for a month

(i) without a break down

(ii) with only one break down

(iii) with at most two break down. **6**

Module – II

8. (a) If $f(x)=\begin{cases} e^{-(x+y)} & x \geq 0,\ y \geq 0 \\ 0 & otherwise \end{cases}$ is a joint probability density function of two dimensional random variable. Find $P\left(\frac{1}{2} < X < 2,\ 0 < Y < 4\right)$. **7**

(b) If $f(x, y) = 2$ for $0 < x < 1, 0 < y < x$ is the joint probability density function of random variables X and Y, find the marginal and conditional density functions. Are X and Y are independent? **7**

(c) Calculate the coefficient of correlation for the following data **6**

x	9	8	7	6	5	4	3	2	1
y	15	16	14	13	11	12	10	8	9

OR

9. (a) Show that $X(t) = A\cos(\omega_0 t + \theta)$ is WSS if A and ω_0 are constants and θ is Uniformly distributed in $(0, 2\pi)$. **10**

(b) Show that the Random Process $X(t) = A\cos\lambda t + B\sin\lambda t$ (A, B are Random variables) is WSS if $E(A) = E(B) = 0$, $E(A^2) = E(B^2)$, $E(AB) = 0$. **10**

Module – III

10. (a) If the auto correlation function of a random process is $R(\tau) = \rho e^{-\rho|\tau|}$, $\rho > 0$, show that the spectral density is given by $S(w) = \dfrac{2}{1+\left(\dfrac{w}{\rho}\right)^2}$. **10**

(b) If the auto covariance function of a stationary process $X(t)$ is given by $C(\tau) = qe^{-\alpha|\tau|}$ ($\alpha > 0$ and q are constants). Show that $X(t)$ is mean ergodic. **10**

OR

11. (a) If the auto correlation function of a WSS process is $R(\tau) = \rho e^{-\rho|\tau|}$, $\rho > 0$, show that $X(t)$ is mean ergodic. **10**

(b) Suppose that customers arrive at a shop in accordance with a Poisson process with mean arrival of 5 minutes. Find the probability that during a time interval of 3 minutes

(i) exactly 10 customers arrive

(ii) more that 10 customers arrive. **10**

Module – IV

12. (a) Find the root between (2, 3) of $x^3 - 2x - 5 = 0$ by regulaFalsi method. **7**

(b) Solve by Gauss Seidal Iteration method

$3x + 2y = 4.5,\ 2x + 3y - z = 5,\ -y + 2z = -0.5$.

Use Initial approximation $x_0 = 0.4,\ y_0 = 1.6,\ z_0 = 0.4$. **7**

(c) Evaluate $\int_0^{\frac{\pi}{2}} \sin x\, dx$ using :

(i) Trapezoidal rule

(ii) Simpson's rule with 10 equal intervals. **6**

OR

13. (a) Solve by Gauss Elimination method :

$$x+2y+z=3,\ 2x+3y+3z=10,\ 3x-y+2z=13.$$ **7**

(b) Using Newton's forward interpolation formula estimate $\sin 47^\circ$ given **7**

θ	45	50	55	60	65
$\sin\theta$	0.7071	0.7660	0.8192	0.8660	0.9036

(c) Using Newton-Raphson's method solve the equation $\cos x + 1 = 3x$ correct to 4 decimal places. **6**