PART – A

Answer all questions. Each question carries 4 marks.

1. If \( f(x) = \frac{k}{2^x} \) is a probability distribution of a random variable which can take values \( x = 0, 1, 2, 3, 4 \). Find \( K \) and Mean of the distribution.

2. Find the mean and variance of the probability distribution with density function 
   \( f(x) = Ke^{-\frac{1}{8}(x^2+8x+16)} \).

3. The customers arrive at a bank according to a Poisson Process with mean rate of 2 minutes. Find the probability that during an 1 minute interval no customers arrive.

4. The autocorrelation function of a stationary process \( \{(X(t))\} \) is given by 
   \( R(\tau) = 2 + 4e^{-2|\tau|} \). Find mean and variance of the process \( \{(X(t))\} \).

P.T.O.
Using Lagrange's interpolation formula find the value of $y$ when $x = 9$ for the following data

\[
\begin{array}{ccc}
X & 5 & 6 & 11 \\
Y & 12 & 13 & 16 \\
\end{array}
\]

\((5 \times 4 = 20 \text{ Marks})\)

PART – B

Answer one full question from each Module. Each question carries 20 marks.

**Module – I**

6. (a) If \( f(x) = \begin{cases} 
0 & x < 2 \\
\frac{1}{18} (2x + 3) & 2 \leq x < 4 \\
0 & x > 4 
\end{cases} \) is the probability density function of a random variable. Find Mean and distribution function.

(b) Human errors is given as the reason for 75% of all accidents in a plant. Use Binomial distribution to find the probability that human error will be given as the reason for 2 of the next 4 accidents.

(c) The mean weight of 500 students at a certain school is 50 kg and the standard deviation is 6 kg. Assuming that the weights are normally distributed, find the expected number of students weighing

(i) between 40 and 50 kg

(ii) more than 60 kg.

OR

\[
2
\]

G – 3586
7. (a) A Random variable has the following probability distribution

\[ X \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \]

\[ f(x) \quad 1/10 \quad k \quad 1/5 \quad 2k \quad 3/10 \quad 3k \]

Find:

(i) \( k \)

(ii) Mean

(iii) Variance

(iv) \( P(-2 < X < 2) \).

(b) The number of cell phones sold daily in a shop is uniformly distributed with a minimum of 50 phones and a maximum of 100 phones. Find the probability that:

(i) the daily sales will fall between 70 and 80 phones

(ii) at least 75 phones are sold on a given day

(iii) at most 70 phones are sold on a given day.

(c) The monthly breakdown of a computer follows Poisson distribution with mean 1.2. Find the probability that this computer will function for a month

(i) without a break down

(ii) with only one break down

(iii) with at most two break down.
Module – II

8. (a) If \( f(x) = \begin{cases} e^{-(x+y)} & x \geq 0, \ y \geq 0 \\ 0 & \text{otherwise} \end{cases} \) is a joint probability density function of two dimensional random variable. Find \( \Pr \left( \frac{1}{2} < X < 2, \ 0 < Y < 4 \right) \).

(b) If \( f(x, y) = 2 \) for \( 0 < x < 1, \ 0 < y < x \) is the joint probability density function of random variables \( X \) and \( Y \), find the marginal and conditional density functions. Are \( X \) and \( Y \) independent?

(c) Calculate the coefficient of correlation for the following data:

<table>
<thead>
<tr>
<th>x</th>
<th>9</th>
<th>8</th>
<th>7</th>
<th>6</th>
<th>5</th>
<th>4</th>
<th>3</th>
<th>2</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>15</td>
<td>16</td>
<td>14</td>
<td>13</td>
<td>11</td>
<td>12</td>
<td>10</td>
<td>8</td>
<td>9</td>
</tr>
</tbody>
</table>

OR

9. (a) Show that \( X(t) = A \cos(\omega_0 t + \theta) \) is WSS if \( A \) and \( \omega_0 \) are constants and \( \theta \) is uniformly distributed in \((0, 2\pi)\).

(b) Show that the random process \( X(t) = A \cos \lambda t + B \sin \lambda t \) (\( A, B \) are random variables) is WSS if \( E(A) = E(B) = 0, \ E(A^2) = E(B^2), \ E(AB) = 0 \).

Module – III

10. (a) If the auto correlation function of a random process is \( R(\tau) = \rho e^{-\rho|\tau|}, \ \rho > 0 \), show that the spectral density is given by \( S(w) = \frac{2}{1 + \left( \frac{w}{\rho} \right)^2} \).

(b) If the auto covariance function of a stationary process \( X(t) \) is given by \( C(\tau) = q e^{-\alpha|\tau|} \) (\( \alpha > 0 \) and \( q \) are constants). Show that \( X(t) \) is mean ergodic.

OR

4

G – 3586
11. (a) If the auto correlation function of a WSS process is \( R(\tau) = \rho e^{-\rho |\tau|}, \rho > 0 \), show that \( X(t) \) is mean ergodic.

(b) Suppose that customers arrive at a shop in accordance with a Poisson process with mean arrival of 5 minutes. Find the probability that during a time interval of 3 minutes

(i) exactly 10 customers arrive

(ii) more that 10 customers arrive.

---

**Module – IV**

12. (a) Find the root between (2, 3) of \( x^3 - 2x - 5 = 0 \) by regulaFalsi method.

(b) Solve by Gauss Seidal Iteration method

\[
3x + 2y = 4.5, \quad 2x + 3y - z = 5, \quad -y + 2z = -0.5.
\]

Use Initial approximation \( x_0 = 0.4, \ y_0 = 1.6, \ z_0 = 0.4 \).

(c) Evaluate \( \int_{0}^{\frac{\pi}{2}} \sin x \, dx \) using:

(i) Trapezoidal rule

(ii) Simpson’s rule with 10 equal intervals.

OR

5

G – 3586
13. (a) Solve by Gauss Elimination method:

\[ x + 2y + z = 3, \ 2x + 3y + 3z = 10, \ 3x - y + 2z = 13. \]

(b) Using Newton’s forward interpolation formula estimate \( \sin 47^\circ \) given

\[
\begin{array}{cccccc}
\theta & 45 & 50 & 55 & 60 & 65 \\
\sin \theta & 0.7071 & 0.7660 & 0.8192 & 0.8660 & 0.9036 \\
\end{array}
\]

(c) Using Newton-Raphson’s method solve the equation \( \cos x + 1 = 3x \) correct to 4 decimal places.