(Pages : 2)

E – 2277

Reg. No. : ....

Name : ....

Third Semester B.Tech. Degree Examination, May 2018
(2013 Scheme)
13.301 : ENGINEERING MATHEMATICS – II (ABCEFHMNPRSTU)

Time : 3 Hours

Max. Marks : 100

PART – A

Answer **all** questions. **Each** question carries **4** marks.

1. Prove that $3y^4z^2\hat{i}+4x^3z^3\hat{j}-3x^2y^2\hat{k}$ is solenoidal.

2. Obtain the Fourier series of $f(x) = x^2, -\pi \le x \le \pi$ and deduce that $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\ldots=\frac{\pi^2}{6}$.

3. Find the Fourier cosine transform of $f(x) = e^{-ax}, 0 \le x < \infty$.

4. Form a partial differential equation by eliminating arbitrary constants from $Z = (x^2 + a^2)(y^2 + b^2)$.

5. Solve the partial differential equation $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=0$ by method of separation of variables.

PART – B

Answer **one full** question from **each** Module. **Each** question carries **20** marks.

MODULE – I

6. a) If $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$ show that (1) $\nabla r=\frac{\vec{r}}{r}$, (2) $\nabla \log r=\frac{\vec{r}}{r^2}$.

   b) Show that $\vec{F}=yz\hat{i}+zx\hat{j}+xy\hat{k}$ is a conservative field and find its scalar potential.

   c) Show that $r^n\vec{r}$ is an irrotational vector for any value of n.

P.T.O.

7. a) Using divergence theorem find $\iint_S \vec{f}\cdot\hat{n}\,ds$, where
$\vec{F} = (x^2 - yz)\hat{i} + (y^2 - zx)\hat{j} + (z^2 - xy)\hat{k}$ over the rectangular parallelopiped $0 \le x \le a,\ 0 \le y \le b,\ 0 \le z \le c$.

b) Find the directional derivative of $\phi = x^2yz + 4xz^2 + xyz$ at (1, 2, 3) in the direction of $2\hat{i} + \hat{j} - \hat{k}$.

c) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ show that $\nabla\cdot\{f(r)\vec{r}\} = rf'(r) + 3f(r)$.

MODULE – II

8. a) Find the Fourier series of the function $f(x) = x + x^2$ in $(-1, 1)$.

b) Find the Fourier sine transform of the function $f(x) = e^{-x} + e^{-2x}$.

9. a) Find the Fourier cosine and Fourier sine series for the function $f(x) = 1,\ 0 < x < L$.

b) Using transforms show that $\int_0^{\infty} \frac{\omega^3 \sin\omega x}{\omega^4 + 4}\,d\omega = \frac{\pi}{2}e^{-x}\cos x$.

MODULE – III

10. a) Solve $x^2(y - z)p + y^2(z - x)q = z^2(x - y)$.

b) Solve $(D - 2D')(D - D'^2)z = e^{x+y}$.

11. a) Solve $p - q = \ln(x + y)$.

b) Solve $(D^2 - 2DD' + D'^2)z = \sin x$.

MODULE – IV

12. a) A tightly stretched string of length L with its fixed ends at x = 0 and x = L executes transverse vibrations. Motion starts with initial velocity zero by displacing the string into the form $f(x) = k(x^2 - x^3)$. Find deflection u(x, t) at any time t.

b) Derive heat equation.

13. a) Find the temperature distribution in a rod of length L whose end points are fixed at temperature zero and initial temperature distribution is f(x).

b) Solve $x\frac{\partial u}{\partial x} - 2y\frac{\partial u}{\partial y} = 0$ using method of separation of variables.
