



Reg. No. :

Name :

**Combined First and Second Semester B.Tech. Degree Examination,
March 2018
(2013 Scheme)**

13.101 : ENGINEERING MATHEMATICS – I (ABCEFHMNPRSTU)

Time : 3 Hours

Max. Marks : 100

Instruction : This question paper has **two** Parts, Part – A and Part – B.

PART – A

Answer **all** questions from this Part. **Each** question carries **4** marks.

1. Find the n^{th} derivative of $y = \cos x \cos 2x \cos 3x$.
2. Evaluate $\int_0^{\infty} \int_0^{\infty} e^{-xy} \sin px \, dx dy$.
3. Find the Laplace transform of unit step function.
4. Find four solutions of the system of equations $x + y + z = 5$, $2x + 2y + 2z = 10$.
5. For what real values of p the system of equations $px + y = 0$, $x - py = 0$ has non-trivial solutions ?

PART – B

This Part has **four** Modules and **each** Module has **two** questions. From **each** Module answer **one full** question. **Each** question carries **20** marks.

Module – I

6. a) If $x = f(y)$ is function satisfying $x^{1/m} + x^{-1/m} = 2y$, show that $(y^2 - 1)x_{n+2} + (2n + 1)yx_{n+1} + (n^2 - m^2)x_n = 0$, where x_n is the n^{th} derivative of x with respect to y .
b) Find the maximum value of the function $f(x, y) = x^3 y^2 (1 - x - y)$.
7. a) Prove that the radius of curvature at any point on the curve $x^{2/3} + y^{2/3} = a^{2/3}$, is three times the length of the perpendicular from the origin to the tangent at that point.

b) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, show that $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z} \right)^2 u = \frac{-9}{(x + y + z)^2}$.

P.T.O.



Module - II

8. a) Find the volume of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ and hence deduce the volume of sphere.

b) Evaluate the integral $\int_0^a \int_{\sqrt{ax}}^a \frac{y^2 dx dy}{\sqrt{y^4 - a^2 x^2}}$ by changing the order of integration.

9. a) Compute the volume integral $\iiint xyz dx dy dz$ over the positive octant of the sphere $x^2 + y^2 + z^2 = 1$.

b) Find the area bounded by the curves $y^2 = 4ax$ and $x^2 = 4by$ and hence deduce the bounded area when $a = b$.

Module - III

10. a) Find the Laplace transform of $\frac{\sin t}{t}$ and hence compute $\int_0^\infty \frac{e^{-4t} \sin t}{t} dt$.

b) Solve $d^4y/dx^4 + 4y = 0$.

11. a) Find the inverse Laplace transform of (i) $\cot^{-1}(3/s)$ and (ii) $\log((s+4)/(s-6))$.

b) Solve the simultaneous differential equations : $dx/dt + y = \sin t$, $dy/dt + x = \cos t$, given $x = 2$, $y = 0$ when $t = 0$.

Module - IV

12. a) Find the rank of the matrix $A = \begin{bmatrix} 1 & 5 & 3 & -2 \\ 2 & -4 & 6 & 2 \\ 5 & 7 & 3 & -9 \\ 3 & 2 & -5 & 6 \end{bmatrix}$.

b) Solve the system equations $kx + y + z = 1$, $x + ky + z = 1$, $x + y + kz = 1$ using the method of rank. Find the value of k when the system is inconsistent.

13. a) Find the orthogonal transformation which will reduce the quadratic form $3x^2 + 5y^2 + 3z^2 - 2zx - 2yz - 2xy$ into canonical form. Find the nature of the quadratic form and its canonical form.

b) If $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$, find the characteristic equation of the matrix

$$B = A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A.$$