Combined First and Second Semester B.Tech. Degree Examination, March 2018
(2013 Scheme)
13.101 : ENGINEERING MATHEMATICS – I (ABCEFHYNPRSTU)

Time : 3 Hours  Max. Marks : 100

Instruction : This question paper has two Parts, Part – A and Part – B.

PART – A

Answer all questions from this Part. Each question carries 4 marks.

1. Find the n<sup>th</sup> derivative of \( y = \cos x \cos 2x \cos 3x \).

2. Evaluate \( \int_{0}^{\infty} \int_{0}^{\infty} e^{-xy} \sin px \, dx \, dy \).

3. Find the Laplace transform of unit step function.

4. Find four solutions of the system of equations \( x + y + z = 5, 2x + 2y + 2z = 10 \).

5. For what real values of \( p \) the system of equations \( px + y = 0, x - py = 0 \) has non-trivial solutions?

PART – B

This Part has four Modules and each Module has two questions. From each Module answer one full question. Each question carries 20 marks.

Module – I

6. a) If \( x = f(y) \) is function satisfying \( x^{1/m} + x^{-1/m} = 2y \), show that
   \[ (y^2 - 1) x_{n+2} + (2n + 1)y x_{n+1} + (n^2 - m^2)x_n = 0, \] where \( x_n \) is the \( n^{th} \) derivative of \( x \) with respect to \( y \).

   b) Find the maximum value of the function \( f(x, y) = x^3 y^2 (1 - x - y) \).

7. a) Prove that the radius of curvature at any point on the curve \( x^{2/3} + y^{2/3} = a^{2/3} \), is three times the length of the perpendicular from the origin to the tangent at that point.

   b) If \( u = \log(x^3 + y^3 + z^3 - 3xyz) \), show that
   \[ \left( \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z} \right)^2 u = \frac{-9}{(x + y + z)^2}. \]

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Module – II

8. a) Find the volume of the ellipsoid \( \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \) and hence deduce the volume of sphere.

b) Evaluate the integral \( \int_0^a \int_0^a \frac{y^2 dx dy}{\sqrt{y^4 - a^2 x^2}} \) by changing the order of integration.

9. a) Compute the volume integral \( \iiint xy z dx dy dz \) over the positive octant of the sphere \( x^2 + y^2 + z^2 = 1 \).

b) Find the area bounded by the curves \( y^2 = 4ax \) and \( x^2 = 4by \) and hence deduce the bounded area when \( a = b \).

Module – III

10. a) Find the Laplace transform of \( \frac{\sin t}{t} \) and hence compute \( \int_0^\infty \frac{e^{-4t} \sin t}{t} dt \).

b) Solve \( d^4 y/dx^4 + 4y = 0 \).

11. a) Find the inverse Laplace transform of (i) \( \cot^{-1}(3/s) \) and (ii) \( \log((s+4)/(s-6)) \).

b) Solve the simultaneous differential equations: \( dx/dt + y = \sin t, dy/dt + x = \cos t \), given \( x = 2, y = 0 \) when \( t = 0 \).

Module – IV

12. a) Find the rank of the matrix \( A = \begin{bmatrix} 1 & 5 & 3 & -2 \\ 2 & -4 & 6 & 2 \\ 5 & 7 & 3 & -9 \\ 3 & 2 & -5 & 6 \end{bmatrix} \).

b) Solve the system equations \( kx + y + z = 1 \), \( x + ky + z = 1 \), \( x + y + kz = 1 \) using the method of rank. Find the value of \( k \) when the system is inconsistent.

13. a) Find the orthogonal transformation which will reduce the quadratic form \( 3x^2 + 5y^2 + 3z^2 - 2zx - 2yz - 2xy \) into canonical form. Find the nature of the quadratic form and its canonical form.

b) If \( A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix} \), find the characteristic equation of the matrix \( B = A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A \).