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Reg. No. : ....

Name : ....

Fifth Semester B.Tech. Degree Examination, January 2018
(2013 Scheme)
13.501 : ENGINEERING MATHEMATICS – IV (AFRT)
(Complex Analysis and Linear Algebra)

Time : 3 Hours Max. Marks : 100

PART – A

Answer **all** questions. **Each** question carries **4** marks.

1. Determine which of the following functions are analytic every where (i) $2xy + i(x^2 - y^2)$ (ii) $(x - iy)/(x^2 + y^2)$.

2. Find the image of the infinite strip $\frac{1}{4} \leq y \leq \frac{1}{2}$ under the transformation $w = \frac{1}{z}$.

3. Evaluate $\int_C \frac{e^{-z}}{z+1} dz$, where C is the circle (a) $|z| = 2$, (b) $|z| = \frac{1}{2}$.

4. Check whether the following subsets of R^3 are subspaces.
   a) The plane of vectors with the first component $b_1 = 0$.
   b) All linear combinations of two given vectors $x = (1, 1, 0)$ and $y = (2, 0, 1)$.

5 Define orthogonal vectors. Give an example.

PART – B

Answer **any one full** question from **each** Module. **Each** question carries **20** marks.

Module – I

6. a) Determine the analytic function $w = u + iv$ if $v = \log(x^2 + y^2) + x - 2y$.
   b) Show that the function $u = e^{-2xy} \sin(x^2 - y^2)$ is harmonic.
   c) Discuss the mapping properties of $w = z^2$. Is it conformal at all points ? Justify your answer.

OR

P.T.O.

7. a) If f(z) is a regular function of z prove that $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4\,|f'(z)|^2$.

b) Find the analytic function whose real part is $3x^2y - y^3$.

c) Show that under the transformation $w = \frac{z-i}{z+i}$, the real axis in the z-plane is mapped into the circle $|w| = 1$.

Module – II

8. a) Expand $f(z) = \frac{(z-1)}{(z+1)}$ in Taylor series about (i) the point z = 0 and about (ii) the point z = 1.

b) Use Cauchy's integral formula to evaluate $\int_C \frac{e^{2z}}{(z-1)(z-2)}\,dz$ where C is the circle $|z| = 3$.

c) Determine the poles of the function $f(z) = \frac{z^2}{(z-1)^2(z+2)}$ and the residue at each pole. Hence evaluate $\int_C f(z)\,dz$ where C is the circle $|z| = 2.5$.

OR

9. a) Evaluate $\int_C \frac{3z^2+z}{z^2-1}\,dz$ where C is the circle $|z-1| = 1$.

b) From the integral $\int_C \frac{dz}{z+2}$ where C is the circle $|z| = 1$, show that

$$\int_0^{\pi} \frac{1+2\cos\theta}{5+4\cos\theta}\,d\theta = 0.$$

c) Expand $f(z) = \frac{1}{(z-1)(z-2)}$ as Laurent's series in the region $1<|z|<2$.

Module – III

10. a) Find the dimension of the null space and column space of the matrix

$$A = \begin{bmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -4 \end{bmatrix}.$$

b) Find a basis for the set of all real 2 × 2 matrices under usual matrix addition and scalar multiplication. Justify your answer.

OR

11. a) Check whether the columns and rows of the matrix $A = \begin{bmatrix} 1 & 3 & 3 & 2 \\ 2 & 6 & 9 & 5 \\ -1 & -3 & 3 & 0 \end{bmatrix}$ are linearly independent.

b) Define the dimension of a vector space. Find the dimension of the subspace $\{(x, y, z) : z = 0\}$ of $\mathbb{R}^3$.

Module – IV

12. a) Find an orthonormal basis for the subspace spanned by (1, 2, 1), (1, 0, 1) and (3, 1, 0) in R^3.

b) Find the least square solution to the inconsistent system AX = b where

$$A = \begin{bmatrix} 1 & 2 \\ 1 & 3 \\ 0 & 0 \end{bmatrix}; b = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}.$$

OR

13. a) Reduce the quadratic form $q = 8x_1^2 + 3x_2^2 + 3x_3^2 + 12x_1x_2 - 8x_2x_3 + 4x_1x_3$ to canonical form by orthogonal transformation.

b) Find a singular value decomposition of $A = \begin{bmatrix} 1 & -1 \\ -2 & 2 \\ 2 & -2 \end{bmatrix}$.