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Reg. No. : ....

Name : ....

First Semester M.Tech. Degree Examination, June 2017

(2013 Scheme)

Branch : ELECTRONICS AND COMMUNICATION ENGINEERING

Stream : Telecommunication

TTM1001 : Linear Algebra

Time : 3 Hours Max. Marks : 60

Instructions : *Answer* ***two full*** *questions from* ***each*** *Module.* ***All*** *questions carry* ***equal*** *marks.*

MODULE – I

I. a) Let (A, $*$) be a group and B be a finite subset of A. Prove that (B, $*$) is a subgroup of A if '$*$' is closed operation in B.

b) A finite group G is of order 13. Find all subgroups of G. Explain your reasoning.

II. a) Define a field and given an example for a finite field with composition tables.

b) If a vector space V has a basis of n elements prove that any set of n+1 vectors of V is linearly dependent.

III. a) Show that the map $T : R^2 \to R^3$ defined by

$T(x_1, x_2) = (2x_1 - x_2, x_1 + x_2, -x_1 + 3x_2)$ is a linear transformation.

b) If $T : R^3 \to R^2$ is defined by $T(x_1, x_2, x_3) = (x_1 + 2x_2 - x_3, -x_1 + 3x_2 + x_3)$, find the matrix of T relative to the bases.

$B = \{x_1 = (1, -2, 3), x_2 = (-2, -2, 0), x_3 = (1, -1, 1)\}$ of R^3, and

$B^1 = \{y_1 = (1, 2), y_2 = (2, 3)\}$ of R^2.

P.T.O.

MODULE – II

IV. a) Find the dimension and basis for the four fundamental subspaces for

$$A = \begin{bmatrix} 1 & 2 & 0 & 2 & 1 \\ -1 & -2 & 1 & 1 & 0 \\ 1 & 2 & -3 & -7 & -2 \end{bmatrix}.$$

b) Solve the system of equations

$8x - 3y - 7w + 2t = 5$

$-9x + 4y + 5w + 11z - 7t = 2$

$6x - 2y + 2z - 4w + 4t = 1$

$5x - y + 7z + 10t = -3.$

V. a) Define a Hilbert space. Illustrate with an example.

b) Let Y be any closed subspace of a Hilbert space H. Prove that $H = Y \oplus Y^{\perp}$.

VI. a) Find an orthonormal basis for the subspace of R^4 spanned by $(1, 1, 1, 1)^T$, $(2, 3, 2, -4)^T$ and $(-1, 5, -2, -1)^T$.

b) Find the equation $y = \beta_0 + \beta_1 x$ of the least squares line that best fits the data points (2, 1), (5, 2), (7, 3) and (8, 3). Also find the least square error.

MODULE – III

VII. a) Find a singular value decomposition of $A = \begin{bmatrix} 1 & -1 \\ -2 & 2 \\ 2 & -2 \end{bmatrix}$.

b) Find a QR factorization of $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$.

VIII. a) Prove that eigen values of a Hermitian matrix are all real and its eigen vectors corresponding to two distinct eigen values are orthogonal.

b) Define circulant matrix and Toeplitz matrices.

IX. a) Derive the DFT of the sample data sequence x(n) = {1, 1, 2, 2, 3, 3}.

b) Find the eigen values and eigen vectors of $A = \begin{bmatrix} 0 & -i & 0 \\ i & 1 & i \\ 0 & -i & 0 \end{bmatrix}$. What property do you expect for the eigen vectors and is it true ?