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C – 2588

Reg. No. : ....

Name : ....

First Semester M.Tech. Degree Examination, June 2017
MECHANICAL ENGINEERING (Machine Design)
(2013 Scheme)
MDC 1003 : Continuum Mechanics

Time : 3 Hours | Total Marks : 60

***Instruction** : Answer **any two** questions from **each Module**.*

MODULE – I

1. a) T is a second order Cartesian tensor. Show that the projection of T onto the orthonormal basis vectors e_i is according to $T_{ij} = e_i \cdot Te_j$ where T_{ij} are the Cartesian components of tensor T. 5

   b) Use the indicial notation to prove the vector identity

   $\nabla \times (u \times v) = v.\nabla u - v\ (\nabla . u) + u(\nabla . v) - u . \nabla v$. 5

2. A deformation of the body is given by $x_1 = -6\,X_2,\ x_2 = \frac{1}{2} X_1,\ x_3 = \frac{1}{3} X_3$ where

   x_i denotes current configuration and X_i denotes reference configuration. The Cauchy stress tensor for a certain point of a body is given by the matrix

   $$[\sigma] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 50 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{kN/cm}^2$$

   Determine the Cauchy traction vector 't' and the first Piola Kirchoff traction vector T acting on a plane having unit outward normal $n = e_2$ in the current configuration. 10

3. The stress tensor at a point P is given by

   $$\sigma_{ij} = \begin{bmatrix} 3 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix}$$

   Determine the principal stress values and the principal stress directions. 10

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MODULE – II

4. Derive an expression for the material time derivative of a smooth spatial field. Apply it to the velocity field of a continuum and thereby explain the concepts of local acceleration and convective acceleration. **10**

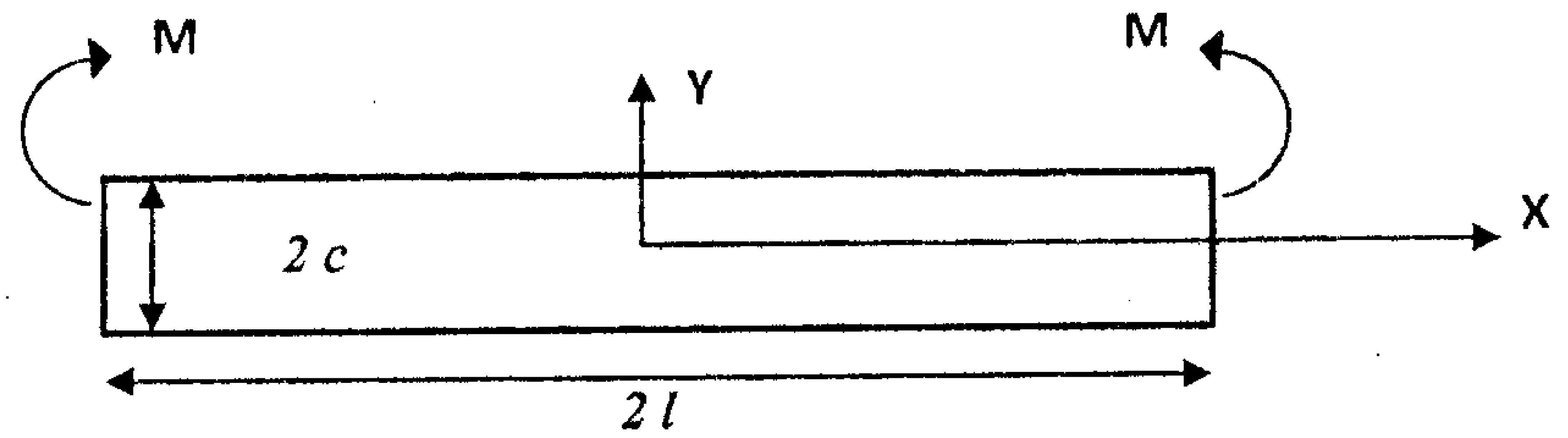
5. Show that the velocity gradient grad $v = \dot{F}\, F^{-1}$ where F is the deformation gradient tensor. **10**

6. State and prove conservation of angular momentum principle and establish the symmetry of the stress tensor. **10**

MODULE – III

7. Show that the Airy stress function $\phi = 2x_1^4 + 12x_1^2x_2^2 - 6x_2^4$ satisfies the biharmonic condition. Also determine the stress components assuming plane strain. **10**

8. For pure bending of a straight beam shown in the figure, find out the stress and the displacement fields. **10**



9. Invert the generalized Hooke's law for isotropic, linear elastic solids as given by $\sigma_{ij} = \lambda\delta_{ij} + 2\mu \in_{ij}$ to get the strains $\in_{ij}$ in terms of engineering constants E and v. For plane stress conditions express $\in_{ij}$ in terms of Lame's constants λ and μ and show that $\in_{33} = -\dfrac{\lambda}{2\mu\,(3\lambda + 2\mu)}\,\sigma_{ii}$. **10**

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