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Reg. No. : ....

Name : ....

First Semester M.Tech. Degree Examination, June 2017
(2013 Scheme)
Branch : Electrical and Electronics Engineering
Streams : Control Systems, Power Control and Drives, Guidance and Navigational Control
EMA1002 : APPLIED MATHEMATICS

Time : 3 Hours Max. Marks : 60

Answer **any two** questions from **each** Module. **All** the modules are **compulsory**. **Each** question carries **10** marks.

MODULE – I

1. a) Define a Vector space. Give one example. Define subspace of a vector space. Show that the set of all (x, y) such that $x + 4y = 0$ form a subspace of $\mathbb{R}^2$.

   b) Find the dimensions of the null and column space of

$$A = \begin{bmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -4 \end{bmatrix}.$$

2. a) Find an orthonormal basis for the solution space of

$$x + y + z = 0$$
$$x - y = 0$$
$$y + 3 = 0.$$

   b) Find the singular value decomposition of $\begin{bmatrix} 2 & 3 \\ 0 & 2 \end{bmatrix}$.

3. a) Consider two bases $\mathscr{B} = \{b_1, b_2\}$ and $\mathscr{C} = \{c_1, c_2\}$ for a vector space. V such that

$$b_1 = 4c_1 + c_2$$
$$\text{and} \quad b_2 = -6c_1 + c_2$$

If $x = 3b_1 + b_2$ find $[x]_{\mathscr{C}}$

P.T.O.

b) Find a QR factorization of

$$\begin{bmatrix} 1 & 2 & 5 \\ -1 & 1 & -4 \\ -1 & 4 & -3 \\ 1 & -4 & 7 \\ 1 & 2 & 1 \end{bmatrix}$$

MODULE – II

4. a) Determine the extremals of the functional $\int_{x_0}^{x_1} (y^2 + y'^2 - 2y\sin x)dx$.

b) Prove that the extremal of the isoperimetric problem.

$V = \int_1^4 y'^2 dx$, y(1) = 3, y(4) = 24

Subject to $\int_1^4 y\, dx = 36$ is a parabola.

5. a) Solve $\frac{dy}{dx} + 4y + 5\int_0^x y(t)dt = e^{-x}$ given y(0) = 0.

b) Solve $y(x) = 1 + \lambda\int_0^1 xt\, y(t)\, dt$ by the method of successive approximation.

6. a) Solve $u_t = 3u_{xx}$ using Laplace transform method given $u\left(\frac{\pi}{2}, t\right) = 0$ $u_x(0, t) = 0$ and the initial condition u (x, 0) = 30 cos 5x.

b) An infinitely long string having one end at x = 0 is initially at rest on X axis. The end x undergoes a periodic transverse displacement described by $A_0 \sin \omega t$, t > 0. Find the displacement of any point on the string at any time t using Laplace transform method.

MODULE – III

7. a) Show that the random process $X(t) = \cos(t + \phi)$ where ϕ is a random variable uniformly distributed in $(0, 2\pi)$ is ergodic based on the first and second order averages.

   b) The process $\{X(t)\}$ is normal with $\mu_t = 0$ and $R_x(\tau) = 4e^{-3|\tau|}$. Find a memory less system g(x) such that the first order density f_y [y] of the resulting output $y(t) = g[X(t)]$ is uniform in the interval (6, 9).

8. a) State and prove Chapman Kolmogorov theorem.

   b) There are 2 white balls in bag A and 3 red balls in bag B. At each step of the process a ball is selected from each bag and the 2 balls selected are interchanged Let the state a_i of the system be the number of red balls in A after i changes. What is the probability that there are 2 red balls in A after 3 steps ? In the long run, what is the probability that there are 2 red balls in A ?

9. a) Obtain formulas for the average number of customers :

      i) in the system

      ii) in the queue and

      iii) in the non empty queues for the (M/M/1) : (∞/FIFO) model.

   b) A travel center has 3 service counters to receive people who visit to book air tickets. The customers arrive in a Poisson distribution with the average arrival of 100 persons in a 10 hour service day. It has been estimated that the service time follows an exponential distribution. The average service time is 15 minutes.

      Find the :

      i) Expected number of customers in the system.

      ii) Expected number of customers in the queue.

      iii) Expected time a customer spends in the system.

      iv) Expected waiting time for a customer in the queue.

      v) Probability that a customer must wait before he gets service.