Combined First and Second Semester B.Tech. Degree Examination, December 2016
(2013 Scheme)
13.101 : ENGINEERING MATHEMATICS – I (ABCEFMNPRSTU)

Time : 3 Hours
Max. Marks : 100

PART – A

Answer all questions. Each question carries 4 marks.

1. If \( u = xyz, v = x^2 + y^2 + z^2, w = x + y + z \) find the Jacobian \( \begin{vmatrix} u, v, w \\ x, y, z \end{vmatrix} \).

2. Evaluate \( \int_0^a \int_0^{\sqrt{a^2 - x^2}} xy \, dy \, dx \).

3. Find \( L^{-1} \left[ \frac{se^{-5t/2} + \pi e^{-s}}{s^2 + \pi^2} \right] \).

4. Show that eigen values of a Hermitian matrix are real.

5. Show that \( 6x_1^2 + 3x_2^2 + 14x_3^2 + 4x_2x_3 + 18x_3x_1 + 4x_1x_2 \) is positive definite.

PART – B

Answer one full question from each Module. Each question carries 20 marks:

MODULE – I

6. a) If \( \cos^{-1} \left( \frac{y}{b} \right) = \log \left( \frac{x}{n} \right)^n \) show that \( x^2y_{n+2} + (2n + 1)xy_{n+1} + 2n^2y_n = 0 \).

b) Find the evolute of the parabola \( x^2 = 4ay \).
7. a) If \( u = \log(x^2 + y^2 + z^2) \), prove that \( u_{xx} + u_{yy} + u_{zz} = \frac{2}{x^2 + y^2 + z^2} \).

   b) Investigate the maximum and minimum values of \( xy (3x + 2y + 1) \).

**MODULE – II**

8. a) Evaluate \( \iint_R xy \, dx \, dy \) where \( R \) is the region bounded by the parabola \( y^2 = x \) and the lines \( y = 0 \) and \( x + y = 2 \) lying in the first quadrant.

   b) Find the area between the circle \( x^2 + y^2 = a^2 \) and the line \( x + y = a \) lying in the first quadrant, by double integration.

9. a) Change the order of integration in \( \int_0^a \int_y^a y \, dx \, dy \) and then evaluate it.

   b) Find the volume bounded by the coordinate planes and the plane \( \frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1 \).

**MODULE – III**

10. a) Using convolution theorem find \( L^{-1} \left[ \frac{S^2}{(S^2 + 4)^2} \right] \)

   b) Solve the differential equation

\[
(2x + 1)^2 y'' - 2 (2x + 1) y' + 2y = 6x.
\]

11. a) Using Laplace transform solve the differential equation \( y'' - 3y' + 2y = 4 \), given that \( y(0) = 2 \), \( y'(0) = 3 \).

   b) Solve the simultaneous differential equations.

\[
\frac{dx}{dt} + 2x - 3y = 5t; \quad \frac{dy}{dt} - 3x + 2y = 2e^{2t}.
\]
MODULE – IV

12. a) Find the rank of the matrix
\[
\begin{bmatrix}
2 & 3 & -1 & -1 \\
1 & -1 & -2 & -4 \\
3 & 1 & 3 & -2 \\
6 & 3 & 0 & -7
\end{bmatrix}
\]

b) Reduce the quadratic form \(6x^2 + 3y^2 + 3z^2 - 4xy - 2yz + 4xz\) into sum of squares. Find the rank, index and signature.

13. a) For what values of \(K\) the equations
\[
\begin{align*}
x + y + z &= 1 \\
2x + y + 4z &= k \\
4x + y + 10z &= k^2
\end{align*}
\]

have a solution and solve them in each case.

b) Find the eigen values and eigen vectors of the matrix
\[
\begin{bmatrix}
8 & -6 & 2 \\
-6 & 7 & -4 \\
2 & -4 & 3
\end{bmatrix}
\]