



Reg. No. :

Name :



**Combined First and Second Semester B.Tech. Degree
Examination, December 2016
(2013 Scheme)**

13.101 : ENGINEERING MATHEMATICS – I (ABCEFHMNPRSTU)

Time : 3 Hours

Max. Marks : 100

PART – A

Answer **all** questions. **Each** question carries **4** marks.

1. If $u = xyz$, $v = x^2 + y^2 + z^2$, $w = x + y + z$ find the Jacobian $J\left(\frac{u, v, w}{x, y, z}\right)$.

2. Evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} xy \, dy \, dx$.

3. Find $L^{-1}\left[\frac{se^{-s/2} + \pi e^{-s}}{s^2 + \pi^2}\right]$.

4. Show that eigen values of a Hermitian matrix are real.

5. Show that $6x_1^2 + 3x_2^2 + 14x_3^2 + 4x_2x_3 + 18x_1x_3 + 4x_1x_2$ is positive definite.

PART – B

Answer **one full** question from **each** Module. **Each** question carries **20** marks:

MODULE – I

6. a) If $\cos^{-1}\left(\frac{y}{b}\right) = \log\left(\frac{x}{n}\right)^n$ show that $x^2y_{n+2} + (2n + 1)xy_{n+1} + 2n^2y_n = 0$.

b) Find the evolute of the parabola $x^2 = 4ay$.

P.T.O.



7. a) If $u = \log(x^2 + y^2 + z^2)$ prove that $u_{xx} + u_{yy} + u_{zz} = \frac{2}{x^2 + y^2 + z^2}$.

b) Investigate the maximum and minimum values of $xy(3x + 2y + 1)$.

MODULE - II

8. a) Evaluate $\iint_R xy \, dx \, dy$, where R is the region bounded by the parabola $y^2 = x$ and the lines $y = 0$ and $x + y = 2$ lying in the first quadrant.

b) Find the area between the circle $x^2 + y^2 = a^2$ and the line $x + y = a$ lying in the first quadrant, by double integration.

9. a) Change the order of integration in $\int_0^a \int_a^{\sqrt{a^2 - y^2}} y \, dx \, dy$ and then evaluate it.

b) Find the volume bounded by the coordinate planes and the plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$.

MODULE - III

10. a) Using convolution theorem find $L^{-1} \left[\frac{S^2}{(S^2 + 4)^2} \right]$

b) Solve the differential equation

$$(2x + 1)^2 y'' - 2(2x + 1)y' + 2y = 6x.$$

11. a) Using Laplace transform solve the differential equation $y'' - 3y' + 2y = 4$, given that $y(0) = 2$, $y'(0) = 3$.

b) Solve the simultaneous differential equations.

$$\frac{dx}{dt} + 2x - 3y = 5t; \quad \frac{dy}{dt} - 3x + 2y = 2e^{2t}.$$



MODULE - IV

12. a) Find the rank of the matrix $\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$

b) Reduce the quadratic form $6x^2 + 3y^2 + 3z^2 - 4xy - 2yz + 4xz$ into sum of squares. Find the rank, index and signature.

13. a) For what values of K the equations

$$x + y + z = 1$$

$$2x + y + 4z = k$$

$$4x + y + 10z = k^2$$

have a solution and solve them in each case.

b) Find the eigen values and eigen vectors of the matrix $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$

