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Reg. No. : ....

Name : ....

Third Semester B.Tech. Degree Examination, October 2016
(2013 Scheme)
13.301 : ENGINEERING MATHEMATICS – II
(ABCEFHMNPRSTU)

Time : 3 Hours Max. Marks : 100

PART – A

(Answer **all** questions. **Each** question carries **4** marks.)

1. Find the directional derivative of $z^2 + 2xy$ at $(1, -1, 3)$ in the direction of $\hat{i} + 2\hat{j} + 2\hat{k}$.

2. Obtain the half range sine series of e^x in $0 < x < 1$.

3. Find the Fourier sine transform of $F(x) = \begin{cases} \sin x , & 0 \le x \le a \\ 0 , & x > a \end{cases}$

4. Obtain the partial differential equation by eliminating arbitrary constants from the relation $z = axe^y + \frac{1}{2}a^2e^{2y} + b$.

5. State the assumptions involved in the derivation of one-dimensional wave equation.

PART – B

(Answer **one full** question from **each** Module. **Each** question carries **20** marks.)

Module – I

6. a) Determine $F(r)$ so that the vector $F(r)\,\vec{r}$ is solenoidal.

P.T.O.

b) Evaluate $\int_C \vec{F}\cdot d\vec{r}$ where 'C' is the circle $x^2 + y^2 = 9$ in the xy plane.

If $\vec{F} = (2x - y + z)\,\hat{i} + (x + y - z^2)\,\hat{j} + (3x - 2y + 4z)\,\hat{k}$.

c) Using Gauss divergence theorem evaluate $\iint_s \vec{F}\cdot d\vec{s}$ where $\vec{F}$ is

$\vec{F} = (2xy + z)\,\hat{i} + y^2\hat{j} - (x + 3y)\,\hat{k}$ and 's' is the surface bounded by $x = 0, y = 0, z = 0$ and $2x + 2y + z = 6$.

7. a) If 's' is a closed surface show that $\iint_s \text{curl}\,\vec{F}\cdot\hat{n}\,ds = 0$.

b) Using Green's theorem in a plane evaluate $\int_c (2x^2 - y^2)\,dx + (x^2 + y^2)\,dy$ where 'C' is the boundary in the xy-plane of the area enclosed by the x-axis and semi-circle $x^2 + y^2 = 1$ in the upper half of the xy-plane.

c) Using Stoke's theorem evaluate $\int_C \vec{F}\cdot d\vec{r}$ where

$\vec{F} = (2x + y - 2z)\,\hat{i} + (2x - 4y + z^2)\,\hat{j} + (x - 2y - z^2)\,\hat{k}$ and 'C' is the circle with centre (0, 0, 3) and radius 5 units in the plane $z = 3$.

Module – II

8. a) Expand x sinx as a Fourier series in $0 < x < 2\pi$.

b) Using Fourier integral show that $\int_0^x \frac{\cos x\lambda}{1+\lambda^2} = \pi/2\, e^{-x}, x \geq 0$.

9. a) Obtain the Fourier series for $f(x) = \begin{cases} 0, & -\pi \leq x \leq 0 \\ \sin x, & 0 \leq x < \pi \end{cases}$ and deduce that

$\frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} \ldots\ldots = \frac{\pi - 2}{4}$.

b) Find the Fourier transform of $f(x) = \begin{cases} 1 - x^2 & \text{if } |x| < 1 \\ 0 & \text{if } |x| > 1 \end{cases}$ use it to evaluate

$\int_0^\infty \frac{x\cos x - \sin x}{x^3} \cos\left(\frac{x}{2}\right) dx$.

Module – III

10. a) Using Charpit's method solve the partial diff. equation $z = px + qy + pq$.

b) Solve the partial differential equation $(D^2 - 2DD' + D'^2)\, z = e^{x+y} x^2y^2$.

11. a) Solve the partial differential equation $y^2p - xyq = x(z - 2y)$.

b) Solve the partial differential equation $(D^2 + D'^2)\, z = \sin 2x \sin 3y + 2\sin^2(x + y)$.

Module – IV

12. a) Solve the equation $\frac{\partial^2 z}{\partial x^2} = \frac{\partial z}{\partial y} + 2z$, by the method of separation of variables given that $z = 0$ and $\frac{\partial z}{\partial x} = 1 + e^{-3y}$ when $x = 0$.

b) A rod of length 20 cm has its ends A and B kept at 30°C and 90°C respectively until steady state conditions prevail. If the temperature at each end is suddenly reduced to 0°C and maintained so, find the temperature u (x, t) at a distance 'x' from A at time t.

13. a) A tightly stretched string of length 'l' has its ends $x = 0$ and $x = l$ fixed. The point $x = l/3$ is drawn aside by a small distance h and released from rest at time $t = 0$, find y (x, t) at any subsequent time t.

b) A rod of length l is heated so that its end A and B are at zero temperature. If initially its temperature is given by $u = cx(l - x)/l^2$. Find the temperature at time t.