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B – 2830

Reg. No. : ....

Name : ....

First Semester M.Tech. Degree Examination, December 2016
(2008 Scheme)
Branch : COMPUTER SCIENCE
RCC 1001 : Mathematical Foundations of Computer Science

Time : 3 Hours Max. Marks : 100

Instruction : *Answer **any five** questions from the **six** questions.*
__Each__ question carries __equal__ marks.

I. a) Using the principle of mathematical induction, prove the correctness of the following pseudo code program segment which is supposed to produce the answer x (y^n) for given real values of x, y and n where n is a non-negative integer :

```
While n ≠ 0 do
    begin
    x : = x*y
    n = n – 1
    end
answer : = x.
```

5

b) Show that from :

i) $(\exists x)(F(x) \wedge S(x)) \rightarrow (\forall x)(M(y) \rightarrow W(y))$ and

ii) $(\exists x)(M(y) \wedge \neg W(y))$

the conclusion $(\forall x)(F(x) \rightarrow \neg S(x))$ follows. **10**

c) Prove the following by contradiction. The number $n^3 - n$ is an even number for any integer n. **5**

P.T.O.

II. a) Solve the recurrence relation $a_{n+2} = 4a_{n+1} - 4a_n$ where n is a positive integer and $a_0 = 1$ and $a_1 = 3$. 6

b) State the principle of inclusion and exclusion. Use this to find the number of positive integers among the first hundred natural numbers which are not divisible by 2, 3 or 5. 6

c) How many strings of 20 decimal digits are there that contain two 0s, four 1s, three 2s, one 3, two 4s, three 5s, two 7s and three 9s ? 8

III. a) Define invertible functions. Show that the function $f(x) = x^3$ is an invertible function. Also find its inverse. 6

b) What do you mean by an equivalence relation ? Define a relation R on Z, the set of integers by xRy (mod n) if and only if n divides x-y where n a fixed positive integer. Show this relation is an equivalence relation. 7

c) A relation R defined on N = {1, 2, 3, 4, 5} is given by R = {(1, 2), (2, 3), (4, 5), (2, 5), (3, 3), (1, 1), (3, 5), (2, 2), (4, 2), (3, 4)}. Find the graphical and Boolean matrix representations of the relation. Also find the Boolean matrix of the relation R°R. 7

IV. a) Define Boolean function. A committee of three members decides issues for a company. Each member votes either yes or no for each proposal that arises. A proposal is passed if it receives at least two yes votes. Construct the Boolean function and a circuit that determines whether a proposal passes. 8

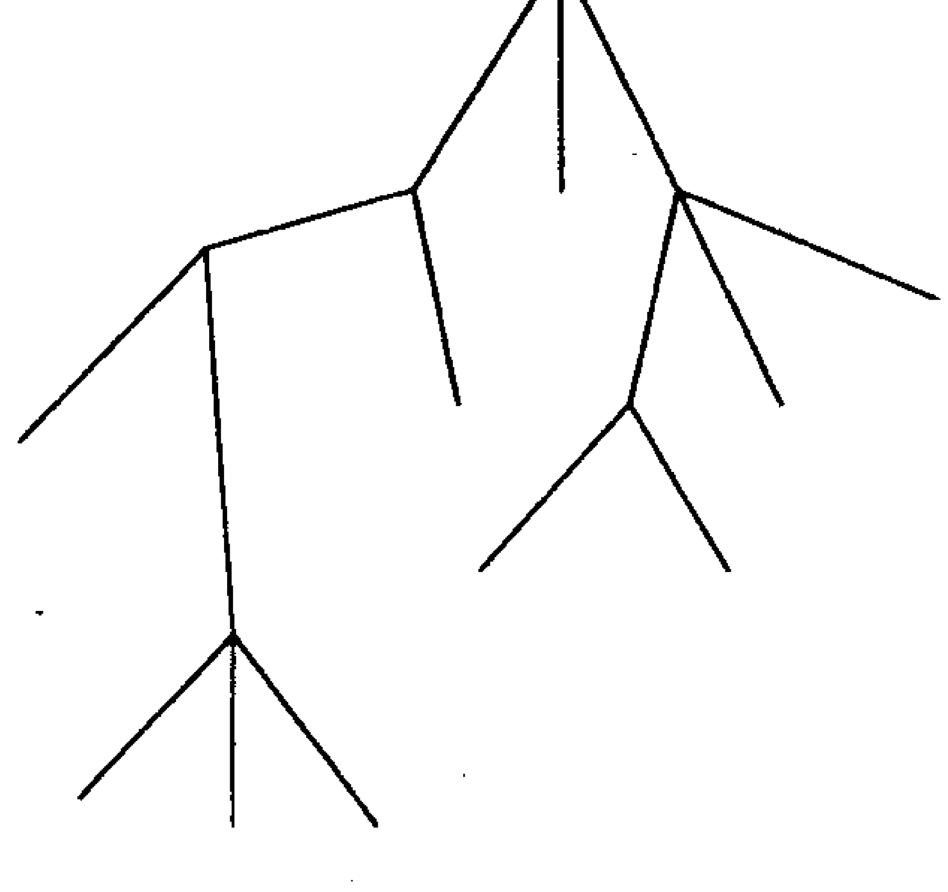
b) Minimize the following Boolean functions :

i) $xy\bar{z} + x\bar{y}\bar{z} + \bar{x}yz + \bar{x}\bar{y}\bar{z}$ and

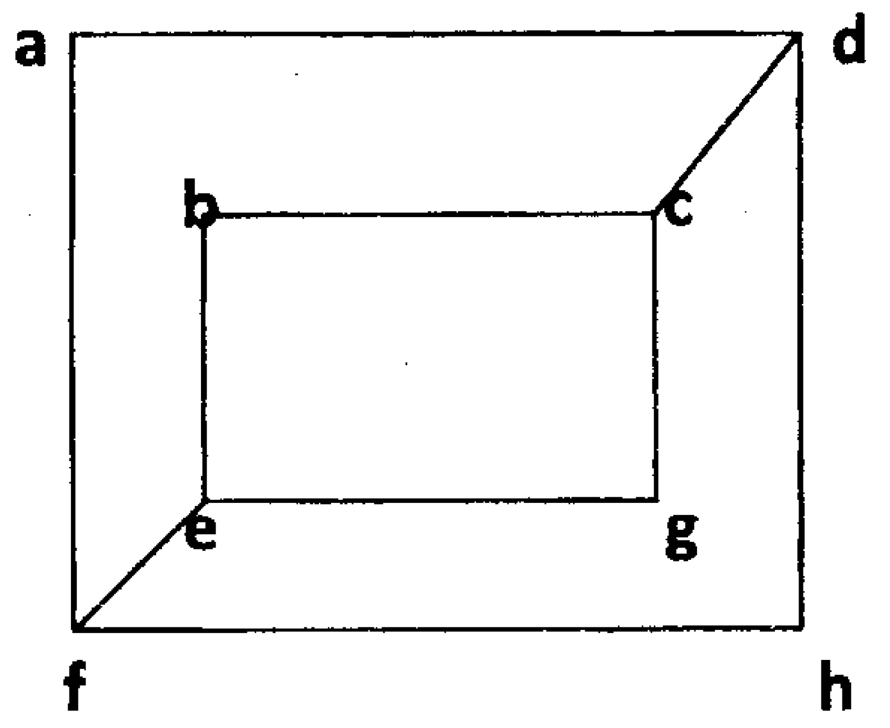
ii) $x\bar{y}\bar{z} + \bar{x}yz + x\bar{y}z + \bar{x}\bar{y}\bar{z} + \bar{x}\bar{y}z$. 6

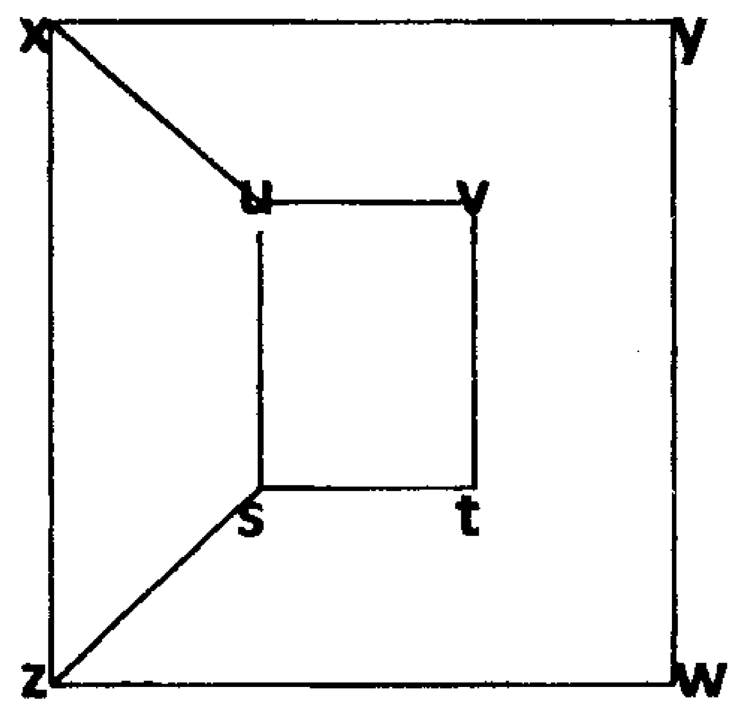
c) State and prove Euler's formula for planar graph. 6

V. a) In which order does a preorder traversal visit the vertices in the ordered rooted tree shown below. (Give proper naming for the 16 vertices of the graph). 8



b) Determine whether the graphs X and Y shown below are isomorphic. **6**





c) Define the following terms :

i) Hamiltonian circuit

ii) Minimal spanning tree

iii) Incidence matrix

iv) Adjacency matrix. **6**

VI. a) Let G be a group and a, b be any two elements in G. Show that : **6**

i) Identity is unique in G and

ii) $(ab)^{-1} = b^{-1}a^{-1}$.

b) Find a transfer sequence from state s_0 to state s_2 for the finite state machine described by the following table : **7**

	ν		ω	
	0	1	0	1
s_0	s_6	s_1	0	1
s_1	s_5	s_0	0	1
s_2	s_1	s_2	0	1
s_3	s_4	s_0	0	1
s_4	s_2	s_1	0	1
s_5	s_3	s_5	1	1
s_6	s_3	s_6	1	1

c) State Kleene's theorem. List the limitations of finite state machines. **7**
