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**B – 3282**

**Reg. No. :** ....................................

**Name :** ..........................................

## Fifth Semester B.Tech. Degree Examination, December 2016
## (2013 Scheme)
## 13.501 : ENGINEERING MATHEMATICS – IV (BCHMPSU)

Time : 3 Hours

Max. Marks : 100

PART – A

Answer **all** questions. **Each** question carries **4** marks.

1. A random variable X has the probability mass function

| **X :** | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| **P(x) :** | 0 | 2k | 3k | k | 2k | $k^2$ | $7k^2$ | $2k^2 + k$ |

   Find the value of k.

2. The probability that an item produced by a machine will be defective is 0.01. Find the probability that a random sample of 100 items selected at random from a total output will contain no more than one defective.

3. A ransom sample of 900 items has a mean 3.7 cms with a standard deviation of 2.61 cms. Is this sample from a large group of mean 3.25 cms and standard deviation 2.61 cms ?

4. Explain :

   i) slack and surplus variables

   ii) degenerate solution in a LPP.

5. Write down the dual of the following LPP,

   Minimize $z = 2x_1 + 3x_2$

   Subject to the constraints, $x_1 + x_2 \geq 10$, $2x_1 + 3x_2 \geq 24$, $x_1 \geq 0$, $x_2 \geq 0$.

P.T.O.

PART – B

Answer **one full** question from **each** Module. **Each** question carries **20** marks.

**Module – 1**

6. a) Derive Poisson distribution as the limiting case of Binomial distribution.

b) In a normal distribution 31% items are under 45 and 8% items are over 64. Find the mean and standard deviation.

c) If a random variable has uniform distribution over (– 3, 3), compute :

i) $P(|X-2|<2)$ and

ii) The value of k for which $P(X > k) = \frac{1}{3}$.

7. a) The time (in years) to the failure of certain components of a system is a random variable having exponential distribution with mean 5. If 5 of these components are in different systems, find the probability that at least 2 are still functioning at the end of 8 years.

b) Given the pdf of the random variable X as

$$f(x) = \begin{cases} kx^2 , & 0 \le x \le 5 \\ 5k(10-x), & 5 < x < 10 \end{cases}$$

Find :

i) the value of k

ii) $P(X < 7)$ and

iii) distribution function $F(x)$.

**Module – 2**

8. a) Calculate the coefficient of correlation between the marks in Economics (X) and marks in Statistics (Y).

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **X :** | 65 | 66 | 67 | 67 | 68 | 69 | 70 | 72 |
| **Y :** | 67 | 68 | 65 | 68 | 72 | 72 | 69 | 71 |

b) 10 bearings made by a process have a mean diameter of 0.5060 cm with a standard deviation of 0.004 cm. If this random sample is from a normal population, find 95% confidence interval for the actual average diameter of the bearings made by this process.

c) In a sample of 1000 people, 540 are rice eaters and the rest are wheat eaters. Can we assume that both rice and wheat are equally popular, at 1% level of significance ?

9. a) Find the lines of regression of X and Y and also the correlation coefficient between X and Y from the following data,

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **X :** | 42 | 44 | 58 | 55 | 89 | 98 | 66 |
| **Y :** | 56 | 49 | 53 | 58 | 65 | 76 | 58 |

b) A sample of 40 bulbs had a mean life time of 647 hours with a standard deviation of 27 hours while another sample of 40 bulbs made by its competitor had a mean lifetime of 638 hours with standard deviation 31 hours. Does this imply any significant difference between the means ? Test at 5% level of significance.

**Module – 3**

10. a) The objective of a diet problem is to ascertain the quantities of certain foods that should be eaten to meet certain nutritional requirements at a minimum cost. The consideration is limited to milk, beef and eggs and to vitamins A, B and C. The number of milligrams of each of these Vitamins contained with a unit of each food are given below.

| Vitamin | Gallon of milk | Pound of beef | Dozens of egg | Minimum daily requirement |
|---|---|---|---|---|
| A | 1 | 1 | 10 | 1 mg |
| B | 100 | 10 | 10 | 50 mg |
| C | 10 | 100 | 10 | 10 mg |
| Cost | Rs.1.00 | Rs.1.10 | Rs.0.50 | |

Formulate a linear programming problem.

b) Solve by simplex method

Minimize $z = x_1 - 3x_2 + 2x_3$

subject to the constraints :

$$3x_1 - x_2 + 2x_3 \leq 7$$
$$-2x_1 + 4x_2 \leq 12$$
$$-4x_1 + 3x_2 + 8x_3 \leq 10$$
$$x_1 \geq 0, x_2 \geq 0\ x_3 \geq 0.$$

11. a) Solve by Big M method

Maximize $z = 6x + 4y$

subject to the constraints :

$$2x + 3y \leq 30$$
$$3x + 2y \leq 24,$$
$$x + y \geq 3$$
$$x \geq 0, y \geq 0.$$

b) Use simplex algorithm to solve the LPP,

Maximize $z = 5x_1 + 3x_2$

subject to

$$x_1 + x_2 \leq 2$$
$$5x_1 + 2x_2 \leq 10$$
$$3x_1 + 8x_2 \leq 12$$
$$x_1 \geq 0, x_2 \geq 0.$$

## Module – 4

12. a) Solve by the principle of duality

Minimize $z = 3x_1 + x_2$

subject to the constraints,

$$x_1 + x_2 \geq 1$$
$$2x_1 + 3x_2 \geq 2$$
$$x_1 \geq 0, x_2 \geq 0.$$

b) Solve the following transportation problem

| | $D_1$ | $D_2$ | $D_3$ | $D_4$ | Capacity |
|---|---|---|---|---|---|
| $O_1$ | 1 | 2 | 3 | 4 | 6 |
| $O_2$ | 4 | 3 | 2 | 0 | 8 |
| $O_3$ | 0 | 2 | 2 | 1 | 10 |
| Demand | 4 | 6 | 8 | 6 | |

13. a) Solve by dual simplex method

Maximize $z = 2x_1 + x_2$

subject to

$$x_1 + 2x_2 \leq 10$$
$$x_1 + x_2 \leq 6$$
$$x_1 - x_2 \leq 2$$
$$x_1 - 2x_2 \leq 1$$
$$x_1 \geq 0, x_2 \geq 0.$$

b) A company has four machines to do three jobs. Each job can be assigned to one and only one machine.

**Machines**

| Jobs | | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| | A | 18 | 24 | 28 | 32 |
| | B | 8 | 13 | 17 | 19 |
| | C | 10 | 15 | 19 | 22 |

What are the job assignments which will minimize the cost ?