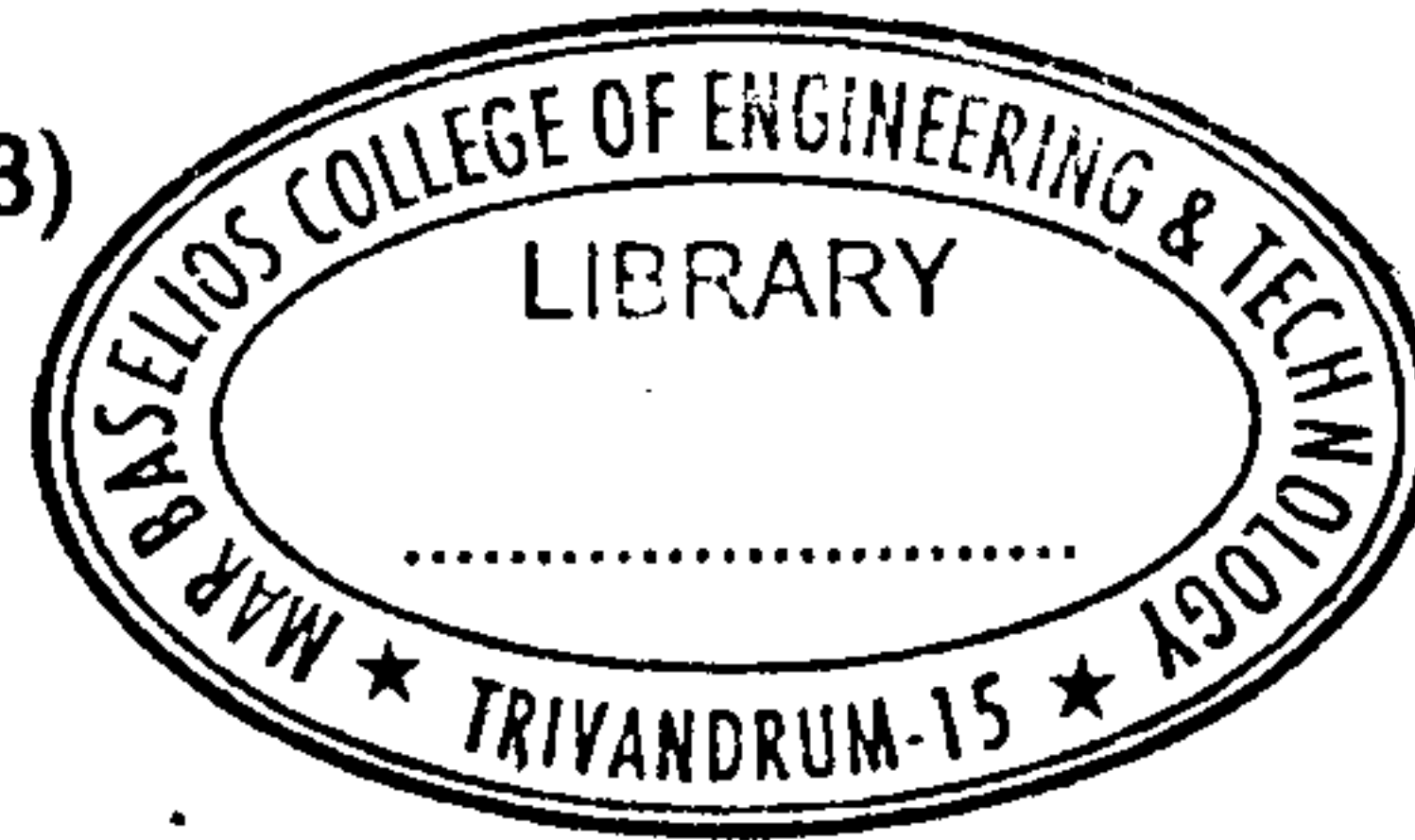




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A – 6482

Reg. No. :

Name :

5. Fifth Semester B.Tech. Degree Examination, October 2016
(2013 Scheme)

13.501 : ENGINEERING MATHEMATICS – IV (AFRT)
(Complex Analysis and Linear Algebra)

Time : 3 Hours

Max. Marks : 100

PART – A

Answer **all** questions. **Each** question carries **4** marks.

1. Prove that the function $f(z) = \sin z$ is analytic and find its derivative.
2. For the conformal transformation $w = z^2$, find the coefficient of magnification and the angle of rotation at $z = 1 + i$.
3. Evaluate $\int_0^{1+i} (x^2 - iy) dz$ along the paths (a) $y = x$, (b) $y = x^2$.
4. Define a vector space. Give an example and justify that your example is a vector space.
5. Find the lengths and inner product of $x = (1, 4, 0, 2)$ and $y = (2, -2, 1, 3)$ with respect to the standard inner product of \mathbb{R}^4 .

PART – B

Answer **any one full** question from **each** Module. **Each** question carries **20** marks.

Module – I

6. a) Show that an analytic function with constant modulus is constant.
b) Show that a complex function which is analytic, at a point satisfies the Cauchy-Riemann equations.
c) Discuss the transformation about $w = e^z$.

OR

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7. a) Determine the analytic function whose real part is $e^{2x} (x \cos 2y - y \sin 2y)$.
- b) If $w = \phi + i \psi$ represents the complex potential for an electric field and $\psi = x^2 - y^2 + \frac{x}{x^2 + y^2}$, determine the function ϕ .
- c) Under the transformation $w = \frac{1}{z}$, find the image of $|z - 2i| = 2$.

Module - II

8. a) Expand the function $f(z) = \cos z$ about $z = \frac{-\pi}{2}$ as Taylor's series.
- b) Evaluate $\int_c \frac{z dx}{(z-1)(z-2)^2}$ where c is the circle $|z-2| = \frac{1}{2}$ using Cauchy's integral formula.
- c) Show that $\int_0^{2\pi} \frac{\cos 2\theta d\theta}{1-2a \cos \theta + a^2} = \frac{2\pi a^2}{1-a^2}, (a^2 < 1)$

OR

9. a) Evaluate $\int_c \frac{z+1}{z^2+2z+4} dz$ where c is $|z+1+i|=2$, using Cauchy's residue theorem.
- b) Use Cauchy's integral formula to calculate $\int_c \frac{z^2+1}{z(2z+1)} dz$ where c is $|z|=1$.
- c) Evaluate $\int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2+1)(x^2+4)}$.



Module - III

10. a) Prove that if v_1, v_2, v_3 are linearly independent, then the vectors $w_1 = v_1 + v_2, w_2 = v_1 + v_3, w_3 = v_2 + v_3$ are also linearly independent.

b) Describe the column space and null space of the matrix $A = \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}$.

OR

11. a) Find the linear transformation $T : \mathbb{R}^2 \rightarrow \mathbb{R}^3$ which takes $(1, 2)$ to $(3, 2, 1)$ and $(3, 4)$ to $(6, 5, 4)$.

b) Examine whether the vectors $(1, 2, 3), (1, -1, 1)$ and $(1, 0, 1)$ form a basis of \mathbb{R}^3 .

Module - IV

12. a) Find a vector x orthogonal to the row space and a vector y orthogonal to the

column space of $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 3 \\ 3 & 6 & 4 \end{bmatrix}$.

b) Find a maxima or minima of $5x_1^2 + 5x_2^2 + 4x_1x_2$ subject to $X^T X = 1$ where

$$X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}.$$

OR

13. Use Gram-Schmidt orthogonalisation to generate a set of orthonormal vectors

from $a_1 = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, a_2 = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}$ and $a_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.

