



Reg. No. :

Name :



**Fifth Semester B.Tech. Degree Examination, September 2016
(2008 Scheme)**

08.502 : ADVANCED MATHEMATICS AND QUEUEING MODELS (RF)

Time : 3 Hours

Max. Marks : 100

Instructions : Answer *all* questions from Part A and *one full* question each from Module I, Module II and Module III.

PART – A

1. Find all basic solutions of the following system.

$$2x_1 + 6x_2 + 2x_3 + x_4 = 3, 6x_1 + 4x_2 + 4x_3 + 6x_4 = 2$$

2. What is simplex method ? Explain the role of artificial variables in Simplex Method.

3. Explain the concepts of slack and float in network analysis. Define total float, free float and independent float.

4. Define the following time estimates

- 1) Earliest start time
- 2) Earliest finish time
- 3) Latest finish time
- 4) Latest start time
- 5) Total project time.

5. Let $A = \begin{bmatrix} -8 & -2 & -9 \\ 6 & 4 & 8 \\ 4 & 0 & 4 \end{bmatrix}$ and $W = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$

Determine if W is in C(A) and N(A).



6. Let $u_1 = (1, 0, 0)$, $u_2 = (-3, 4, 0)$ and $u_3 = (3, -6, 3)$ show that $B = \{u_1, u_2, u_3\}$ is a basis of \mathbb{R}^3 . Find the coordinate vector for $V = (-8, 2, 3)$.
7. Find the conic $ax^2 + by^2 = 1$ that best fits the points $(1, 1)$, $(0, 2)$, $(1, -1)$ and $(-1, 2)$.
8. Distinguish between transient and steady states.
9. Explain the characteristics of queueing models.
10. Derive the formulae for the average number of customers in the queueing system.
(4×10=40 Marks)

PART – B

Module – I

11. Solve the following LPP

$$\text{Max } z = 4x_1 + 5x_2 + 2x_3$$

$$\text{Subject to } 2x_1 + x_2 + x_3 \leq 10,$$

$$x_1 + 3x_2 + x_3 \leq 12$$

$$x_1 + x_2 + x_3 = 6 \quad x_1, x_2, x_3 \geq 0.$$

12. Construct the network for the project whose activities are given below and compute the total, free and independent float of each activity and hence determine the critical path and project duration.

Activity	Duration (in weeks)	Activity	Duration (in weeks)
1 – 2	8	3 – 4	3
1 – 3	7	3 – 5	5
1 – 5	12	3 – 6	10
2 – 3	4	4 – 6	7
2 – 4	10	5 – 6	4

**Module – II**

13. a) Solve the following equation $AX = B$ by using LU factorization of A.

$$x_1 - 2x_2 - 4x_3 - 3x_4 = 1$$

$$2x_1 - 7x_2 - 7x_3 - 6x_4 = 7$$

$$-x_1 + 2x_2 + 6x_3 + 4x_4 = 0$$

$$-4x_1 - x_2 + 9x_3 + 8x_4 = 3$$

- b) Find an orthonormal basis for the subspace spanned by $(1, 0, 1, 2)$, $(2, 1, 0, 2)$ and $(1, -1, 0, 1)$ in R^4 .
14. a) Show that $B = \{(2, 1, -1), (-1, 1, -1), (0, 3, 3)\}$ forms an orthogonal basis for R^3 . Also find the coordinates of an arbitrary vector in R^3 relative to the basis B .

- b) Find a singular value decomposition of the matrix $A = \begin{bmatrix} 1 & -1 \\ -2 & 2 \\ 2 & -2 \end{bmatrix}$.

Module – III

15. a) A T.V. repairman finds that the time spent on his job has an exponential distribution with mean 30 minutes. If he repairs sets in the order in which they came in and if the arrival of sets is Poisson with an average rate of 10 per 8 hour day, what is his expected idle time day? How many jobs are ahead of the average set just brought in?
- b) Customers arrive at a window drive of a bank as per Poisson law with mean 10 per hour. Service time per customer is exponential with mean 5 minutes. The space can accommodate 4 cars including the car serviced.
- What is the probability that the arriving car can drive directly to the space?
 - What is the probability that an arriving car will have to wait?
 - How long an arriving car is expected to wait before starting service?



16. A post office has 3 windows providing the same services. It receives on an average 30 customers/hour. Arrivals are Poisson distributed and service time exponentially distributed. The post office serves on an average 12 customers/hour

- a) What is the probability that a customer will be served immediately ?
- b) What is the probability that a customer will have to wait ?
- c) What is the average number of customers in the system ?
- d) What is the average time that a customer must spend in the post office ?
- e) What is the average queue length ?
- f) Find the average number of idle servers.

(3×20=60 Marks)

