



Reg. No. : .....

Name : .....

SECOND SEMESTER B.TECH. DEGREE EXAMINATION, MAY/JUNE 2016

**MA 102 : DIFFERENTIAL EQUATIONS**

Max. Marks : 100

Duration : 3 Hours

## PART – A

Answer **all** questions and **each** question carries **3** marks.

1. Determine a linearly independent solution of the differential equation  $(x^2 + 1) y'' - 2xy' + 2y = 0$  if  $y_1 = x$  is solution.
2. Solve the differential equation  $y^{IV} + 6y''' + 9y'' = 0$ .
3. Find the particular integral of the differential equation  $(D^2 - 2D + 1)y = xe^x$ .
4. Solve by the method of variation parameters,  $(D^2 + 4)y = \tan 2x$ .
5. Develop the Fourier series of  $f(x) = x^2$  in  $-2 \leq x \leq 2$ .
6. Find the Fourier sine series of  $f(x) = e^x$  in  $0 < x < 1$ .
7. Obtain the partial differential equation by eliminating  $f$  and  $g$  from  $z = xf(y) + yg(x)$ .
8. Solve the partial differential equation  $(y^2 + z^2)p - xyq + xz = 0$ .
9. Obtain the solution of the wave equation  $\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 u}{\partial t^2}$  using method of separation of variables when the separation constant  $k < 0$ .
10. Write any two assumptions involved in deriving one dimensional wave equation.
11. Find the steady state temperature distribution in a rod of length 20 cm if the ends of the rod are kept at  $10^\circ \text{C}$  and  $70^\circ \text{C}$ .
12. Solve  $\frac{\partial u}{\partial t} = h \frac{\partial^2 u}{\partial x^2}$  subject to the conditions  $u(0, t) = u(1, t) = 0$  for  $t > 0$  and  $u(x, 0) = 3 \sin n\pi x$ ,  $0 < x < 1$ .

**(12×3=36 Marks)**



## PART – B

Answer **six** questions – **one full** question from **each** Module.

## Module – 1

13. a) Reduce to first order and hence solve the ODE

i)  $y'' + (y')^3 \cos y = 0$  and

ii)  $2xy'' = 3y'$ .

b) Solve the IVP  $y'' - 2y' + 5y = 0$ ,  $y(0) = -3$ ,  $y'(0) = 1$ .

11

OR

14. a) Show that the functions  $x$  and  $x \ln(x)$  are linearly independent (use Wronskian). Hence form an ODE for the given basis  $x$ ,  $x \ln(x)$ .

b) Solve the IV  $Py'' + 0.2y' + 4.01y = 0$ ,  $y(0) = 0$ ,  $y'(0) = 2$ .

11

## Module – 2

15. a) Solve the differential equation  $(D + 1)^2y = x^2e^x$ .

b) Solve the differential equation  $(x^3D^3 + 3x^2D^2 + xD + 1)y = x + \log x$ .

11

OR

16. a) Solve the differential equation  $(D^2 + 1)y = x^2e^x + \sin x$ .

b) Solve the differential equation  $(x + 1)^2y'' + (x + 1)y' - y = 2 \sin \log(x + 1)$ .

11

## Module – 3

17. a) Find the Fourier Series of  $f(x) = \begin{cases} x & , 0 < x < 1 \\ 1-x & , 1 < x < 2 \end{cases}$

b) Find the Fourier cosine series of  $f(x) = x(\pi - x)$  in  $0 < x < \pi$ .

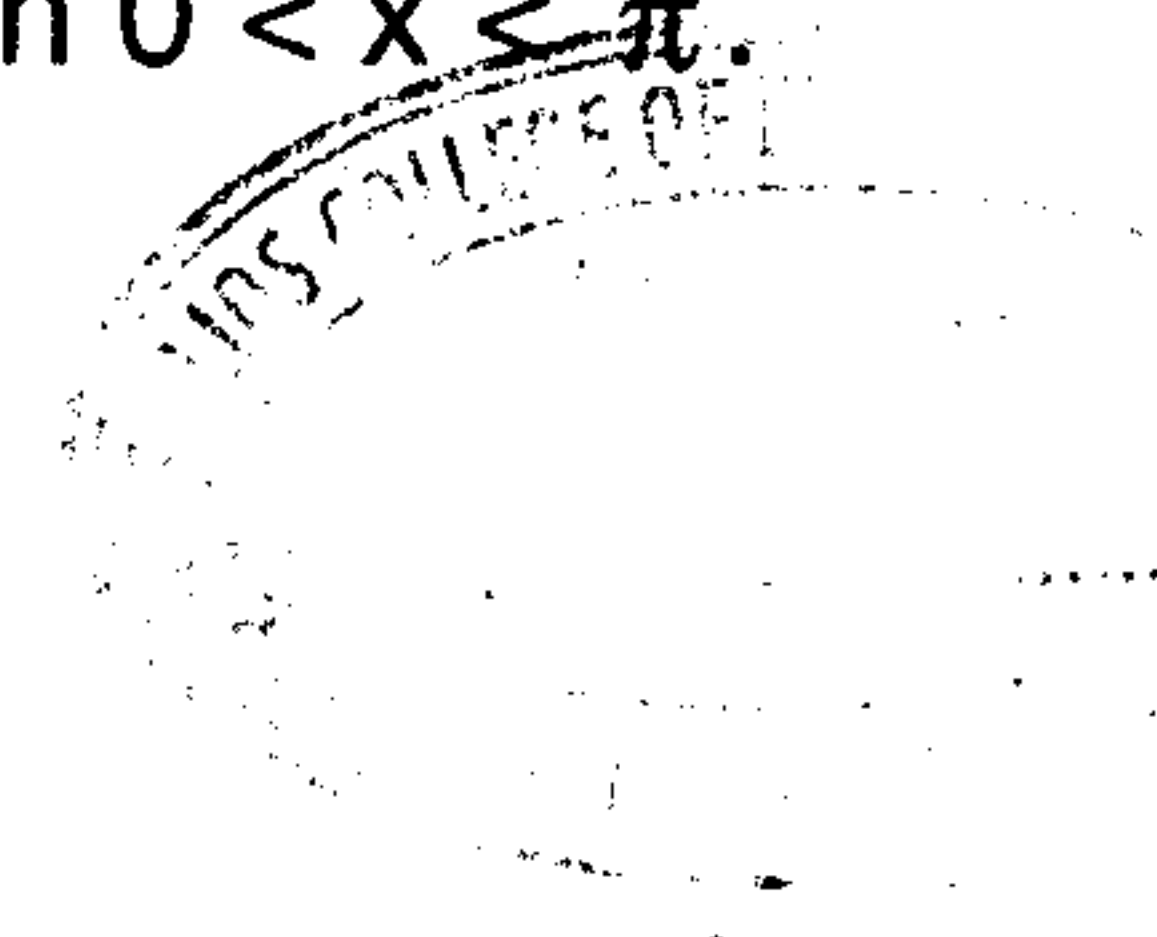
11

OR

18. a) Expand  $f(x) = e^{-x}$  in  $(-l, l)$  as a Fourier Series.

b) Find the half range sine series of  $f(x) = x \sin x$  in  $0 < x < \pi$ .

11



**Module – 4**

19. a) Form the PDE by eliminating a, b, c from  $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ .

b) Solve the partial differential equation  $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 2 \frac{\partial^2 z}{\partial y^2} = e^{2x+y}$  11

OR

20. a) Solve :  $x(y^2 - z^2)p + y(z^2 - x^2)q = z(x^2 - y^2)$ .

b) Solve the partial differential equation  $\frac{\partial^3 z}{\partial x^3} - 4 \frac{\partial^3 z}{\partial^2 x \partial y} + 4 \frac{\partial^3 z}{\partial x \partial y^2} = \cos(2x + y)$ . 11

**Module – 5**

21. A tightly stretched string of length 'a' with fixed ends is initially in equilibrium position. Find the displacement  $u(x, t)$  of the string if it is set vibrating by giving each of its points a velocity  $v_0 \sin(\pi x/a)$ . 10

OR

22. A transversely vibrating string of length 'a' is stretched between two points A and B. The initial displacement of each point of the string is zero and the initial velocity at a distance x from A is  $kx(a - x)$ . Find the form of the string at any subsequent time. 10

**Module – 6**

23. Find the temperature in a laterally insulated bar of length L whose ends are kept at temperature zero if the initial temperature is  $f(x) = \begin{cases} x & , 0 < x < L/2 \\ L - x & , L/2 < x < L \end{cases}$ . 10

OR

24. An insulated rod of length L has its ends A and B maintained at  $0^\circ \text{C}$  and  $100^\circ \text{C}$  respectively until steady state conditions prevails. If B is suddenly reduced to  $0^\circ \text{C}$  and maintained at  $0^\circ \text{C}$ , then find the temperature in the rod at a distance x from A at time t. 10

