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Reg. No. : ....

Name : ....

Fifth Semester B.Tech. Degree Examination, December 2015
(2008 Scheme)
08.501 – ENGINEERING MATHEMATICS – IV
Complex Analysis and Linear Algebra (TA)

Time : 3 Hours Max. Marks : 100

Instructions : *Answer **all** questions from Part **A** and **one full** question from **each** Module of Part **B**.*

PART – A **(10×4 = 40 Marks)**

1. Show that $f(z) = \frac{1}{z}, z \neq 0$ is differentiable every where except z = 0 and find its derivative.

2. If u and v are harmonic functions, prove that $\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right) + i\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)$ is analytic.

3. Prove that an analytic function with a constant modulus is a constant.

4. Find the image of $|z - 3i| = 3$ under $w = \frac{1}{z}$.

5. Show that $\int_c \frac{1}{z} dz = \pi i$ or $-\pi i$ according as C is the semicircle $|z| = 1$ above or below the real axis from (1,0) to (–1, 0).

6. Expand $\frac{z}{z+2}$ about z = 1. State the region of validity.

7. Evaluate $\int_c \frac{\sin^2 z}{(z - \pi/6)^2} dz$ where C is $|z| = 1$.

P.T.O.

8. Define subspace of a vector space.

   Show that $H = \{(a-b, b-c, c-a, b)^T : a, b, c \in R\}$ is subspace of R^4.

9. Show that $u_1 = (1, 0, 0)$, $u_2 = (-3, 4, 0)$ and $u_3 = (3, -6, 3)$ is a basis of R^3.

10. Find the conic $ax^2 + by^2 = 1$ that best fits the points (1,1), (0,2), (–1, 1) and (–1,2).

PART – B **(3×20 = 60 Marks)**

Module – I

11. a) Prove that $f(z) = \dfrac{x^3(1+i) - y^3(1-i)}{x^2 + y^2}$, $z \neq 0$ and $f(0) = 0$ statistics CR equations at the origin, but $f'(0)$ does not exist.

    b) Find the analytic function $f(z) = u + iv$ if $v = x^3y - xy^3 + xy + x + y$.

    c) Discuss the transformation $w = \sin z$.

12. a) Find the analytic function $f(z) = u + iv$ if $u - v = e^x(\cos y - \sin y)$.

    b) If f(z) is analytic show that $\left(\dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2}\right) \log |f'(z)|^2 = 0$.

    c) Find the bilinear transformation which maps (1, i, –1) into (i, –1, –i)

Module – II

13. a) Evaluate $\int_c \dfrac{z\,dz}{(4-z^2)(z+1)}$ where C is $|z| = 3$.

    b) Find the poles and residues of $\dfrac{z^2 + 2z}{(z+1)^2(z^2+4)}$.

    c) Find the Laurent's series expansion of $f(z) = \dfrac{z^2 - 1}{(z+2)(z+3)}$ in $2 < |z| < 3$.

14. a) Evaluate $\int_0^{2\pi} \frac{d\theta}{5+4\sin\theta}$ by contour integration in the complex plane.

b) Show that $\int_0^{\infty} \frac{dx}{1+x^6} = \frac{\pi}{3}$.

Module – III

15. a) Solve the following equation AX = B by using L U factorization of A

$x + 3y + 4z = 1$

$-3x - 6y - 7z + 2t = -2$

$3x + 3y - 4t = -1$

$-5x - 3y + 2z + 9t = 2.$

b) Find the dimension of C(A), R(A) and N(A)

if $A = \begin{bmatrix} 1 & 3 & 1 & -2 \\ 1 & 4 & 3 & -1 \\ 2 & 3 & -4 & -7 \\ 3 & 8 & 1 & -7 \end{bmatrix}$

c) Find an orthonormal basis from the basis
{(1, 1, 1), (0, 1, 1), (0, 0, 1)}.

16. a) Find the nature of the quadratic form

$Q(x) = 3x_1^2 + 2x_2^2 + x_3^2 + 4x_1 x_2 + 4x_2 x_3$.

b) Find the singular value decomposition of $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 0 & 0 \end{bmatrix}$

c) Find the projection matrix of $u = (1, 2, 3)^T$ and hence find the projection of $v = (1, -1, 1)$ on u.

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