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Reg. No. : ....

Name : ....

Combined First and Second Semester B.Tech. Degree Examination, April 2016 (2008 Scheme)

08-101 : ENGINEERING MATHEMATICS – I

Time : 3 Hours Max. Marks : 100

PART – A

Answer **all** questions. **Each** question carries **4** marks.

1. Find the n^{th} derivative of sin x sin 2x sin 3x.

2. If $y = (\sin^{-1}x)^2$ show that $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - n^2y_n = 0$.

3. Find J and $J^{\star}$ if $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$ and $z = r\cos\theta$.

4. Prove that curl grad $\phi = 0$.

5. Solve $\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = 0$.

6. Find :

   i) $L\left[\cos^2(2t - 1)\right]$

   ii) $L\left[e^{-2t}u(t - 1)\right]$.

7. If L[f(t)] = F(s), prove that $L\left[f(at)\right] = \frac{1}{a}F\left[\frac{s}{a}\right]$.

P.T.O.

8. Determine the rank of the matrix $A = \begin{bmatrix} 1 & 1 & -1 & 1 \\ 1 & -1 & 2 & -1 \\ 3 & 1 & 0 & 1 \end{bmatrix}$ by reducing into echelon form.

9. Check whether the vectors $X_1 = (1, 2, 4)$, $X_2 = (2, -1, 3)$, $X_3 = (0, 1, 2)$ and $X_4 = (-3, 7, 2)$ are linearly dependent. If so find a relation connecting them.

10. If $\lambda \neq 0$ is an eigen value of A, then prove that $\frac{1}{\lambda}$ is an eigen value of A^{-1}.

PART – B

Answer **two** questions from **each** Module. **Each** question carries **10** marks.

Module – I

11. a) Find the evolute of $x = a\cos^3\theta$, $y = a\sin^3\theta$.

    b) If $u = \sin^{-1}\left(\frac{y}{x}\right)$ prove that $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$.

12. a) Find the percentage error in the area of an ellipse if 2% error is made in measuring the major and minor axes.

    b) Expand $e^x \cos y$ as a Taylor series about the point $\left(1, \frac{\pi}{4}\right)$.

13. a) Find the directional derivative of $u = 2xy + z^2$ at the point $(1, -1, 3)$ in the direction of $i + 2j + 2k$.

    b) Show that $\vec{f} = (y\sin z - \sin x)i + (x\sin z + 2yz)j + (xy\cos z + y^2)k$ is irrotational. Find its scalar potential.

Module – II

14. a) Show that the family of curves $y^2 = 4c(c + x)$ is self orthogonal.

b) Solve $(D^2 - 2D + 1)y = xe^x \sin x$.

15. a) Solve $(D^2 - 2D)y = e^x \sin x$ by method of variation of parameters.

b) Reduce the following equation to a linear differential equation with constant coefficients

$$(1+x)^2 \frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = 2\sin[\log(1+x)].$$

16. Solve $(D^3 - 3D^2 + 3D - 1)y = t^2e^t$ given that $y(0) = 1,\ y'(0) = 0,\ y''(0) = -2$ by the method of Laplace transforms.

Module – III

17. a) Use Gauss-Jordan method to find the inverse of $\begin{bmatrix} 2 & 1 & -1 \\ 0 & 2 & 1 \\ 5 & 2 & -3 \end{bmatrix}$.

b) Find the value of λ and μ such that $x + 2y + \lambda z = 1$, $x + 2\lambda y + z = \mu$, $\lambda x + 2y + z = 1$ have unique solution.

18. a) Find the eigen values and eigen vectors of $\begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$.

b) Use Cayley-Hamilton theorem to find A^3 given $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$.

19. Reduce the quadratic form $4x^2 + 3y^2 + z^2 - 8xy - 6yz + 4xz$ into a canonical form by an orthogonal transformation and give the matrix of transformation. Also state the nature of the quadratic form.