(Pages : 2) **9197**

Reg. No. : ....

Name : ....

First Semester M.Tech. Degree Examination, February 2015
(2013 Scheme)
Electronics and Communication – Telecommunication Engineering
TTM 1001 : LINEAR ALGEBRA

Time : 3 Hours Max. Marks : 60

***Instruction**: Answer **two full** questions from **each** Module.*

Module – I

1. a) Prove that, for a vector space V over the field F, a non empty subset W of V is a subspace of V if and only if for each pair of vectors α, β in W and each scalar C in F, the vector $C\alpha + \beta$ is again in W.

   b) Prove that the intersection of any collection of subspaces of a vector space V over the field F is a subspace of V.

2. If W_1 and W_2 are finite dimensional subspaces of a vector space V, then show that $W_1 + W_2$ is finite dimensional and
   $\dim W_1 + \dim W_2 = \dim (W_1 \cap W_2) + \dim (W_1 + W_2)$.

3. a) Find the null space, range, rank and nullity of the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^3$ defined by $T(x, y) = (x + y, x - y, y)$.

   b) Define linear functional on a vector space V and prove that trace function of an $n \times n$ matrix A is a linear functional on the matrix space $F^{n\times n}$.

Module – II

4. Find all least square solutions X in $\mathbb{R}^3$ of A X = b where

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 2 \\ -1 & 1 & -1 \\ -1 & 2 & 0 \end{bmatrix} \qquad b = \begin{bmatrix} 3 \\ -3 \\ 0 \\ -3 \end{bmatrix}$$

P.T.O.

5. a) Explain Gram-Schmidt orthogonalization process.

   b) Prove that every finite dimensional inner product space has an orthonormal basis.

6. Let W be a finite dimensional subspace of an inner product space V and let E be the orthogonal projection of V on W. Then, prove that, E is an idempotent linear transformation of V onto W, $W^{\perp}$ is the null space of E and $V = W \oplus W^{\perp}$.

Module – III

7. a) Show that the eigen values of a Hermitian matrix are real.

   b) Diagonalize $\begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$.

8. a) Find singular value decomposition of $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$.

   b) Define circulant matrix. Write down the 3 by 3 circulant matrix

   $C = 2I + 5P + 4P^2$ where $P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

   Find its eigen values.

9. a) Find QR factorization of $A = \begin{bmatrix} 2 & 2 \\ 1 & 7 \\ -2 & -8 \end{bmatrix}$.

   b) Define Toeplitz matrix with an example.
