Instruction: Answer any two questions from each Module.

Module – I

1. a) Derive the relation between input and output spectra for an M-fold decimator. Illustrate with an example.  
   
   b) For the system shown in the Fig. below, find simplified expression for \( y(n) \) in terms of \( x(n) \).

\[
\begin{array}{c}
\uparrow 3 \\
x(n) \\
\downarrow 2 \\
\ \ \\
\downarrow 3 \\
\ \ \\
\uparrow 2 \\
\downarrow \ \\
y(n)
\end{array}
\]

2. a) Derive output \( Y(z) \) of the system shown below in terms of the polyphase components of \( H(z) \).

\[
\begin{array}{c}
\uparrow M \\
x(n) \\
\ \ \\
\ \ \\
H(z) \\
\downarrow M \\
\ \ \\
y(n)
\end{array}
\]

b) Develop a two band polyphase decomposition of the following transfer function

\[
\frac{2 + 3.1z^{-1} + 1.5z^{-2}}{1 + 0.9z^{-1} + 0.8z^{-2}}
\]

3. What is the necessary and sufficient condition for perfect reconstruction? Show that M-channel uniform DFT filter bank is a perfect reconstruction system. Also find the synthesis filters in the DFT filter bank.
Module – II

4. Given \( \Psi(t) = \begin{cases} 
1, & 0 \leq t < \frac{1}{2} \\
-1, & \frac{1}{2} \leq t < 1 \\
0, & \text{otherwise} 
\end{cases} \)

and \( \psi_{m,n}(t) = 2^{-m/2} \psi(2^{-m} t - n), m, n \in \mathbb{Z} \).

Prove that the set of functions \( \{ \psi_{m,n}(t) \}_{m,n \in \mathbb{Z}} \) forms an orthogonal basis for \( L_2(\mathbb{R}) \).

5. Describe the necessary conditions to be considered for deriving orthogonal wavelet system coefficients.

6. Describe digital filter implementation of Haar wavelet decomposition.

Module – III

7. Give the advantages of biorthogonal wavelet system over orthogonal system. Also explain signal representation using biorthogonal wavelet system.

8. Derive Daubechies wavelet system with three vanishing moment (a 6-tap wavelet system) using frequency domain approach.

9. Explain in details the application of wavelets in denoising.