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Reg. No. : ....

Name : ....

First Semester M.Tech. Degree Examination, February 2015
(2013 Scheme)
Branch : Mechanical Engineering
Stream : Machine Design
MDM 1001 : ENGINEERING MATHEMATICS

Time : 3 Hours Max. Marks : 60

***Instruction**: Answer **any two** questions from **each** module.*

Module I

1. a) If $\vec{r} = xi + yj + zk$, then show that $\text{div}\left(\text{grad}\left(\frac{1}{r}\right)\right) = 0$ where $r = |\vec{r}|$.

   b) Apply Green's theorem to evaluate $\int (2x^2 - y^2)\,dx + (x^2 + y^2)\,dy$ where C is the boundary of the area enclosed by the x-axis and the upper half of the circle $x^2 + y^2 = a^2$.

2. Verify Stokes theorem for $\vec{F} = (y - z + 2)i + (yz + 4)j - xzk$ for the surface of the box bounded by the planes $x = 0$, $y = 0$, $z = 0$, $x = 1$, $y = 2$, $z = 3$ above the xy plane.

3. a) A covariant tensor has components xy, $2y - z^2$, xz in rectangular coordinates. Find its covariant components in spherical coordinates.

   b) State and prove any one property of eigen vectors of tensors.

Module II

4. a) Solve the integral equation $y(x) = x + 2\int_0^x \cos(x - t)\,y(t)\,dt$ using transforms.

   b) Solve by using method of successive approximation,

   $y(x) = 1 + x + \int_0^x (x - t)\,y(t)\,dt$.

P.T.O.

5. a) Examine whether the equation $y(1 + z^2)\,dx - x\,(1 + z^2)\,dy + (x^2 + y^2)\,dz = 0$ is integrable or not. If so find its integral.

   b) Reduce the equation $3\frac{\partial^2 u}{\partial x^2} + 10\frac{\partial^2 u}{\partial x \partial y} + 3\frac{\partial^2 u}{\partial y^2} = 0$ to canonical form and solve it.

6. Consider the initial value problem for a string located at position $y(x, t = 0) = y_0(x)$ as a function of distance along the string x and vertical speed $\left(\frac{\partial y}{\partial t}\right)t = 0 = V_0(x)$. Find the resultant motion.

Module III

7. a) Express $\int_0^1 \frac{dx}{\sqrt{(1-x^4)}}$ in terms of Gamma function.

   b) Prove that $\int_{-1}^{1} x^2 P_{n-1} P_{n+1}\,dx = \frac{2n(n+1)}{(2n-1)(2n+1)(2n+3)}$.

8. Consider a laterally insulated metal bar of length 1 units whose ends are kept at zero degree temperature and the temperature at $t = 0$ is $f(x) = \sin \pi x$. Apply Crank-Nicolson method to solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ with $h = 0.2$ and $r = 1$. Find the temperature $u(x, t)$ in the bar for $0 \leq t \leq 0.2$.

9. Solve using finite difference method, the equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ on a square region of dimension 12 cm with boundary conditions $u(x, 0) = 100$, $0 \leq x \leq 12$, $u(x, 12) = 0$, $0 \leq x \leq 12$, $u(0, y) = 100$, $0 \leq y \leq 12$, $u(12, y) = 100$, $0 \leq y \leq 12$. Use grid size of dimension 4 cm.