Reg. No. : ....

Name : ....

First Semester M.Tech. Degree Examination, February 2015
(2013 Scheme)
Branch : Mechanical Engineering
Stream : Machine Design
MDC 1003 : CONTINUUM MECHANICS

Time : 3 Hours Max. Marks : 60

***Instructions** : Answer **any two full** questions from **each** Module. **All** question carries **10** marks.*

MODULE – I

1. The stress tensor values at a point P are given by the array

$$\sigma_{ij} = \begin{pmatrix} 1 & -3 & \sqrt{2} \\ -3 & 1 & -\sqrt{2} \\ \sqrt{2} & -\sqrt{2} & 4 \end{pmatrix}.$$ Determine

a) the principle stress values σ_I, σ_{II} and σ_{III}, together with the corresponding principal stress directions.

b) the stress invariants I_I, I_{II} and I_{III}.

c) the maximum shear stress value and the normal to the plane on which it acts.

d) the stress vector on the octahedral plane together with its normal and shear components.

10

2. a) Using index notation prove the vector identities.

i) (a × b) . (c × d) = (a . c) (b . d) – (a . d) (b . c)

ii) curl (u × v) = (div v) u – (div u) v + (v . ∇) u + (u. ∇) v.

(2×4=8 Marks)

P.T.O.

b) Evaluate

i) $\delta_{ip}\ \delta_{jq}\ a_p\ b_j\ c_q$

ii) $\epsilon_{pqr}\ \epsilon_{rqp}$. **(2×1=2 Marks)**

3. a) The equilibrium configuration of a body is described by

$x_1 = 16X_1,\ x_2 = -\frac{1}{2}X_2,\ x_3 = -\frac{1}{4}X_3$ and the Cauchy stress tensor is given by $T_{11} = 1000\text{MPa}$ and all other $T_{ij} = 0$.

i) Calculate the first Piola-Kirchohoff stress tensor and the corresponding pseudo-stress vector for the plane whose undeformed plane is the e_1-plane.

ii) Calculate the second Piola Kirchhoff tensor and the corresponding pseudo-stress vector for the same plane. **6**

b) The stress matrix representation at P in MPa is given by

$$\sigma_{ij} = \begin{pmatrix} 29 & 0 & 0 \\ 0 & -26 & 6 \\ 0 & 6 & 9 \end{pmatrix}$$

Decompose this matrix into its spherical and deviator parts and determine the principal deviator stress values. **4**

MODULE – II

4. a) For the homogeneous deformation expressed by the equations

$x_1 = \sqrt{2}X_1 + \frac{3\sqrt{2}}{4}X_2,\ x_2 = -X_1 + \frac{3}{4}X_2 + \frac{\sqrt{2}}{4}X_3,\ x_3 = X_1 - \frac{3}{4}X_2 + \frac{\sqrt{2}}{4}X_3.$

Calculate. **7**

i) The unit normal $\hat{n}$ for the line element originally in the direction of $\hat{N} = (\hat{I}_1 - \hat{I}_2 + \hat{I}_3)/\sqrt{3}$.

ii) The stretch $\Lambda_{\hat{N}}$ of this element.

iii) The maximum and minimum stretches at the point $X_1 = 1, X_2 = 0, X_3 = -2$ in the reference configuration.

b) If the motion of a continuous medium is given by

$x_1 = X_{1}e^{t} - X_3(e^t - 1), x_2 = X_{2}e^{-t} + X_3(1 - e^{-t}, x_3 = X_3$. Determine the displacement field in both material and spatial descriptions. 3

5\. a) Given the deformation defined by 8

$x_1 = \alpha X_1 + \beta X_2, x_2 = -\alpha X_1 + \beta X_2, x_3 = \mu X_3$ where α, β and μ are constants.

i) The magnitude and directions of the principal stretches.

ii) The matrix representation of the rotation tensor R.

iii) The direction of the axis of the rotation vector and the magnitude of the angle of rotation.

iv) The right and left stretch tensor.

b) Prove that $C = I + \nabla u + \nabla u^T + (\nabla u^T)(\nabla u)$. 2

6\. a) State and explain the conservation of linear momentum. Obtain the Eulerian form of the conservation of linear momentum. 7

b) Obtain the spatial form of the continuity equation from the material form. 3

MODULE – III

7\. a) Using the Hooke's law of a linear elastic isotropic solid show that

$\sigma_{ij} = (3\lambda + 2\mu)\varepsilon_{ii}$ and using the result deduce that $\varepsilon = \frac{1}{2\mu}\left[\sigma_{ij} - \frac{\lambda}{3\lambda + 2\mu}\delta_{ij}\,\sigma_{kk}\right]$.

6

b) For a linearised elastic isotropic material, obtain the wave equation in 2D, from the strain-displacement relation and the constitutive equation. 4

8. Use the stress function $\phi = (Ar^2 + Br^4 + C/r^2 + D)\cos 2\theta$ to solve the stress problem of a flat plate under a uniform axial stress S, and having a small circular hole at the origin as shown in figure 1. **10**

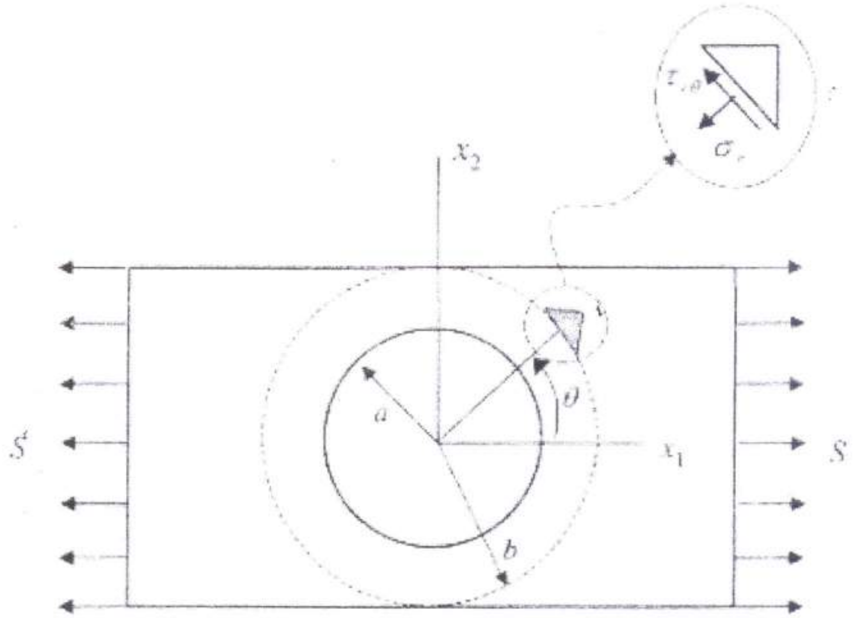

Figure 1

9. Show that $\phi = x_1^4 x_2 + 4x_1^2 x_2^3 - x_2^5$ is a valid Airy stress function and compute the stress tensor for this case, assuming a state of plane strain with $\nu = 0.25$. **10**