Reg. No. : ....

Name : ....

First Semester M.Tech. Degree Examination, February 2015
(2013 Scheme)
Branch : Electrical and Electronics Engineering
Streams : Control Systems Power Control and Devices, Guidance and Navigational Control
EMA – 1002 : APPLIED MATHEMATICS

Time : 3 Hours Max. Marks : 60

Answer **any two** questions from **each** Module. **All** questions carry **equal** marks.

Module – I

1. a) Define null space and column space of an mxn matrix. Show that they are subspaces of R^n and R^m respectively.

   b) Find the dimension of the subspace $H = \left\{ \begin{bmatrix} a-4b-2c \\ 2a+5b-4c \\ -a+2c \\ -3a+7b+6c \end{bmatrix} : a, b, c \in R \right\}$ of a vector space.

2. a) Suppose $T : U \to V$ is a linear transformation. Show that $\text{Im}(T) = \{T(u), u \in U\}$ is a subspace of V.

   b) Find the QR factorization of $A = \begin{bmatrix} 1 & 3 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$.

3. Find the singular value decomposition of $A = \begin{bmatrix} 4 & 11 & 14 \\ 8 & 7 & -2 \end{bmatrix}$.

P.T.O.

Module – II

4. a) Find the curve on which the functional $\int_0^{\frac{\pi}{2}} (y'^2 - y^2 + 2xy)\,dx$ with $y(0) = 0$ and $y'\left(\frac{\pi}{2}\right) = -1$ is an extremum.

b) Find the extremal of the functional $\int_0^{\pi/2} ((y')^2 + (z')^2 + 2yz)dx$ given that $y(0) = 0$, $y(\pi/2) = -1$, $z(0) = 0$, $z(\pi/2) = 1$.

5. a) Solve $y(x) = 3x^2 + \int_0^x \sin(x-t)\,y(t)\,dt$.

b) Show that $y(x) = 1 + \int_0^x (x+t)\,y(t)\,dt$ is equivalent to $y''(x) - 2xy'(x) - 3y(x) = 0$, $y(0) = 1$, $y'(0) = 0$.

6. Solve the boundary value problem $u_t = u_{xx}$ given that $u(\pi/2, t) = 0$, $u_t(0, t) = 0$; $u(x, 0) = 30 \cos 5x$, using Laplace transform method.

Module – III

7. a) On day 0, a house has two new light bulbs in reserve. The probability that the house will need a single new light bulb during day n is p and the probability that the house will not need any is $q = 1 - p$. Let Y_n be the number of light bulbs left in the house at the end of day n.Is $\{Y_n\}$ a Markov Chain ? Find the transition probability matrix.

b) A Markov model for speech assumes that if the nth packet contains silence, then the probability of silence in the next packet is $1 - a$ and the probability of speech activity is a. Similarly, if the nth packet contains speech activity, the probability of speech activity in the next packet is $1 - b$ and the probability of silence is b. Let X_n denote speech activity in a packet at time n. Find the steady state distribution of the chain.

8. a) Draw the state transition diagram and classify the states of a Markov chain having states 0, 1, 2, 3 and with the following tpm $A = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0.5 & 0.5 & 0 \end{bmatrix}$

   Is the chain Ergodic ? Why ?

   b) Suppose that children are born in a hospital at a Poisson rate of 5/day. What is the probability that

      i) at least 2 babies are born during the next 6 hours

      ii) no babies are born during the next two days.

9. A road transport company has two reservation clerks serving the customers. The customers arrive in a Poisson fashion at the rate of 8/hour. The service time for each customer is exponentially distributed with mean 10 minutes. Find the

   i) Probability that a customer has to wait for service

   ii) Average number of customers in the queue

   iii) Average number of customers in the system

   iv) Expected waiting time of a customer in the queue

   v) Expected time a customer spends in the system.
