



Reg. No. :

Name :

**First Semester M.Tech. Degree Examination, February 2015
(2013 Scheme)**

Branch : Electrical and Electronics Engineering

**Streams : Control Systems Power Control and Devices, Guidance and
Navigational Control**

EMA – 1002 : APPLIED MATHEMATICS

Time : 3 Hours

Max. Marks : 60

Answer **any two** questions from **each** Module. **All** questions carry **equal** marks.

Module – I

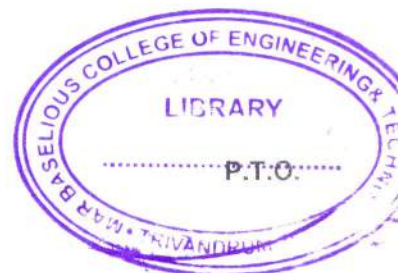
1. a) Define null space and column space of an $m \times n$ matrix. Show that they are subspaces of R^n and R^m respectively.

- b) Find the dimension of the subspace $H = \left\{ \begin{bmatrix} a - 4b - 2c \\ 2a + 5b - 4c \\ -a + 2c \\ -3a + 7b + 6c \end{bmatrix} : a, b, c \in R \right\}$ of a vector space.

2. a) Suppose $T : U \rightarrow V$ is a linear transformation. Show that $\text{Im}(T) = \{T(u), u \in U\}$ is a subspace of V .

- b) Find the QR factorization of $A = \begin{bmatrix} 1 & 3 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$.

3. Find the singular value decomposition of $A = \begin{bmatrix} 4 & 11 & 14 \\ 8 & 7 & -2 \end{bmatrix}$.





Module – II

4. a) Find the curve on which the functional $\int_0^{\frac{\pi}{2}} (y'^2 - y^2 + 2xy) dx$ with $y(0) = 0$ and

$$y'\left(\frac{\pi}{2}\right) = -1 \text{ is an extremum.}$$

b) Find the extremal of the functional $\int_0^{\pi/2} ((y')^2 + (z')^2 + 2yz) dx$ given that $y(0) = 0$, $y(\pi/2) = -1$, $z(0) = 0$, $z(\pi/2) = 1$.

5. a) Solve $y(x) = 3x^2 + \int_0^x \sin(x-t)y(t) dt$.

b) Show that $y(x) = 1 + \int_0^x (x+t)y(t) dt$ is equivalent to $y''(x) - 2xy'(x) - 3y(x) = 0$, $y(0) = 1$, $y'(0) = 0$.

6. Solve the boundary value problem $u_t = u_{xx}$ given that $u(\pi/2, t) = 0$, $u_t(0, t) = 0$; $u(x, 0) = 30 \cos 5x$, using Laplace transform method.

Module – III

7. a) On day 0, a house has two new light bulbs in reserve. The probability that the house will need a single new light bulb during day n is p and the probability that the house will not need any is $q = 1 - p$. Let Y_n be the number of light bulbs left in the house at the end of day n . Is $\{Y_n\}$ a Markov Chain? Find the transition probability matrix.

b) A Markov model for speech assumes that if the n th packet contains silence, then the probability of silence in the next packet is $1 - a$ and the probability of speech activity is a . Similarly, if the n th packet contains speech activity, the probability of speech activity in the next packet is $1 - b$ and the probability of silence is b . Let X_n denote speech activity in a packet at time n . Find the steady state distribution of the chain.



8. a) Draw the state transition diagram and classify the states of a Markov chain

having states 0, 1, 2, 3 and with the following tpm $A = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0.5 & 0.5 & 0 \end{bmatrix}$

Is the chain Ergodic ? Why ?

- b) Suppose that children are born in a hospital at a Poisson rate of 5/day. What is the probability that
- i) at least 2 babies are born during the next 6 hours
 - ii) no babies are born during the next two days.
9. A road transport company has two reservation clerks serving the customers. The customers arrive in a Poisson fashion at the rate of 8/hour. The service time for each customer is exponentially distributed with mean 10 minutes. Find the
- i) Probability that a customer has to wait for service
 - ii) Average number of customers in the queue
 - iii) Average number of customers in the system
 - iv) Expected waiting time of a customer in the queue
 - v) Expected time a customer spends in the system.

