



Reg. No. : .....

Name : .....

**Third Semester B.Tech. Degree Examination, November 2014  
(2013 Scheme)**

**13.303 : DISCRETE STRUCTURES (FR)**

Time : 3 Hours

Max. Marks : 100

PART – A

Answer **all** questions. **Each** question carries **2** marks.

1. Show that  $\neg(P \wedge Q) \rightarrow (\neg P \vee (\neg P \vee Q)) \Leftrightarrow (\neg P \vee Q)$ .
2. What is indirect method of Proof ? Using the same show that  $\neg(P \wedge Q)$  follows from  $\neg P \wedge \neg Q$ .
3. Differentiate between a statement and a statement function.
4. If a Relation 'R' is transitive, then prove that its inverse  $R^{-1}$  is also transitive.
5. Let  $P = \{2, 3, 6, 12, 24, 36\}$ . The relation " $\leq$ " be such that  $x \leq y$ ; if  $x$  divides  $y$ . Draw the Hasse diagram of  $(P, \leq)$ .
6. Define the symmetric difference between two sets.
7. Prove that the inverse of an element in a group is unique.
8. What is meant by Ring with zero divisors ? Explain.
9. Differentiate between simple and elementary path in a graph.
10. Prove that, in a distributive lattice, if an element has complement then this complement is unique.

**(10x2=20 Marks)**

P.T.O.



## PART – B

Answer **any one** question from **each** Module.

**Module – I**

11. a) Without constructing the truth table, prove that

$$(\neg P \wedge (\neg Q \wedge R)) \vee (Q \wedge R) \vee (P \wedge R) \Leftrightarrow R. \quad 7$$

b) Using indirect method, prove that

$$S \rightarrow \neg Q, S \vee R, \neg R, \neg R \leftrightarrow Q \Rightarrow \neg P. \quad 7$$

c) Check the validity of the following statements

“All integers are rational numbers

Some integers are powers of 2.

Therefore, some rational numbers are powers of 2”.

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12. a) Show that

$$((P \vee Q) \wedge \neg(\neg P \wedge (\neg Q \vee \neg R))) \vee (\neg P \wedge \neg Q) \vee (\neg P \wedge \neg R) \text{ is a tautology.} \quad 10$$

b) Show that from

i)  $(\exists x)[F(x) \wedge S(x)] \rightarrow \forall y[M(y) \rightarrow W(y)]$

ii)  $(\exists y)[M(y) \wedge \neg W(y)]$ , the conclusion

$$(\forall x)(F(x) \rightarrow \neg S(x)) \text{ follows.} \quad 10$$

**Module – II**

13. a) Construct a formula for the sum of first 'n' positive odd numbers. Prove the same using mathematical induction. 5

b) Let R and S be two relations on a set of positive integers I.

$$R = \{ \langle x, 2x \rangle / x \in I \} \quad S = \{ \langle x, 7x \rangle / x \in I \}. \text{ Find } R \circ S, R \circ R, R \circ R \circ R, R \circ S \circ R. \quad 10$$

c) For any two sets A and B, show that  $A - (A \cap B) = A - B$ . 5



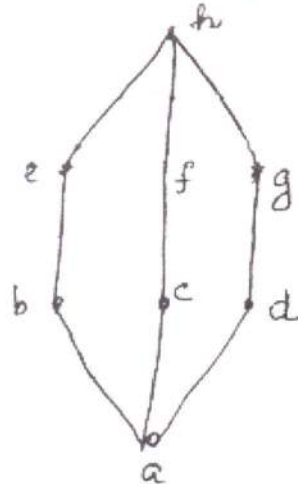
- 14. a) Prove that if  $n \geq 1$ , then  $1(1!) + 2(2!) + \dots + n(n!) = (n+1)! - 1$ . 5
- b) Show that countable union of countable sets is countable. 5
- c) Let 'N' be the set of natural numbers. 'R' is a relation defined on the set  $N \times N$  of ordered pairs defined by  $(a, b) R (c, d)$  if  $ad = bc$ . Prove that 'R' is an equivalence relation. 10

**Module – III**

- 15. a) Show that the set 'N' of natural numbers for the composition  $a \circ b = a + b + ab$  is a semigroup. Is it a Monoid ? 8
- b) Prove that every subgroup of an abelian group is a normal subgroup. 5
- c) Show that if  $a, b \in G$ , then  $(ab)^2 = a^2b^2$  if 'G' is abelian. 7
- 16. a) State and prove Lagrange's theorem. 10
- b) Discuss different algebraic systems with two binary operations with examples. 10

**Module – IV**

- 17. a) Show that every chain is a distributive lattice. 8
- b) Differentiate between a Boolean function and Boolean expression. 2
- c) Explain connected, disconnected and strongly connected graphs using examples. 10
- 18. a) Find all sunlattices of the given lattice. 10



- b) Let  $x, y$  be arbitrary elements in a Boolean algebra  $(B, +, \cdot, 1, 0)$ . Prove the Demorgan's laws.  
 $(x + y)' = x'y'$  and  $(xy)' = x' + y'$  10