Combined First and Second Semester B.Tech. Degree Examination, October 2014
(2013 Scheme)
13.101 : ENGINEERING MATHEMATICS – I (ABCEFHMNRSTU)

Time : 3 Hours

Max. Marks : 100

PART – A

Answer all questions. Each question carries 4 marks.

1. Find the radius of curvature at the point (−4, 0) on the curve xy² = 16(x+4).

2. Evaluate \( \int_{0}^{1} \int_{0}^{y} \int_{0}^{z} \) dzdydx.

3. Using method of variation of parameters solve the differential equation
   \( \frac{d^2y}{dx^2} + a^2y = \sec ax. \)

4. Show that the vectors (1, 3, 4, 2), (3, −5, 2, 2) and (2, −1, 3, 2) are linearly
dependent. Express one vector as a linear combination of others.

5. If \( \lambda = 1 \) is an eigen value of
   \[
   \begin{pmatrix}
   2 & 1 & 1 \\
   1 & 2 & 1 \\
   1 & 1 & 2
   \end{pmatrix}
   \]
   Find the other eigen values.

PART – B

Answer one full question from each Module. Each question carries 20 marks.

MODULE – I

6. a) Find the values of ‘a’ and ‘b’ such that \( \lim_{\theta \to 0} \frac{\theta (1 + a \cos \theta) - b \sin \theta}{\theta^3} = 1 \)

   b) Find the minimum value of \( x^2 + y^2 + z^2 \) subject to the condition \( xyz = a^3. \)
7. a) If $z = F(x, y)$, $x = e^u \cos v$, $y = e^u \sin v$ show that $x \frac{\partial F}{\partial v} + y \frac{\partial F}{\partial u} = e^{2u} \frac{\partial F}{\partial y}$

b) Show that the evolute of the curve $x = a \left( \cos t + \log \tan \frac{t}{2} \right)$; $y = a \sin t$ is $y = a \cosh \frac{x}{a}$.

MODULE II

8. a) Evaluate $\int \int_{0}^{\sqrt{1-x^2}} y^2 \, dy \, dx$ by changing the order of integration.

b) Find the area lying between the parabola $y = 4x - x^2$ and the line $y = x$.

9. a) Evaluate $\int_{R} \int x^2 \, dy \, dx$ where ‘$R$’ is the region bounded by the curves $y = x$ and $y = x^2$.

b) Find the volume bounded by the cylinder $x^2 + y^2 = 4$ the planes $y + z = 3$ and $z = 0$.

MODULE III

10. a) i) Find the Laplace transform of $te^{2t} \sin 5t$.

ii) Find $L^{-1} \left( \frac{s^2 + 2}{s(s^2 + 4)} \right)$.

b) Solve the differential equation $(D^2 - 4D + 1)y = e^{2x} \sin 2x$.

11. a) i) Find $L \left( \frac{1 - \cos t}{t} \right)$.

ii) Find the Laplace transform of the periodic function $F(t)$ of period ‘$a$’ defined by $F(t) = \begin{cases} 1 & \text{for } 0 < t < \frac{a}{2} \\ -1 & \text{for } \frac{a}{2} < t < a \end{cases}$
b) Solve the differential equation
\[ x^2 \frac{d^2 y}{dx^2} - x \frac{dy}{dx} + 4y = \cos (\log x) + x \sin (\log x). \]

**MODULE - IV**

12. a) Find the rank of the matrix.

\[
\begin{bmatrix}
1 & 2 & 3 & 4 & 5 \\
6 & 7 & 8 & 9 & 10 \\
11 & 12 & 13 & 14 & 15 \\
16 & 17 & 18 & 19 & 20 \\
21 & 22 & 23 & 24 & 25
\end{bmatrix}
\]

b) Diagonalise the matrix

\[
\begin{bmatrix}
1 & 1 & 3 \\
1 & 5 & 1 \\
3 & 1 & 1
\end{bmatrix}
\]

13. a) Find the eigen values and eigen vectors of

\[
\begin{bmatrix}
6 & -2 & 2 \\
-2 & 3 & -1 \\
2 & -1 & 3
\end{bmatrix}
\]

b) Solve the system of equations.

\[
\begin{align*}
2x_1 + x_2 + 2x_3 + x_4 &= 6 \\
6x_1 - 6x_2 + 6x_3 + 12x_4 &= 36 \\
4x_1 + 3x_2 + 3x_3 - 3x_4 &= -1 \\
2x_1 + 2x_2 - x_3 + x_4 &= 10
\end{align*}
\]