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Reg. No. : ....

Name : ....

Fifth Semester B.Tech. Degree Examination, May 2012
(2008 Scheme)
08.501: ENGINEERING MATHEMATICS – IV
Complex Analysis and Linear Algebra (T A)

Time : 3 Hours Max. Marks : 100

Instructions : *Answer **all** questions from Part **A** and **one full** question from **each** Module of Part **B**.*

PART – A

1. Prove that $f(z) = \frac{z-1}{z+1}$ is differentiable at every point $z \neq -1$ and find $f'(z)$. **(40)**
2. Show that : $u(x, y) = x^2 - y^2$ and $v(x, y) = \frac{-y}{x^2+y^2}$ are both harmonic but u+iv is not analytic.
3. Find the values of the constants a, b, c, d such that the function $f(z) = x^2 + axy + by^2 + i(cx^2 + dxy + y^2)$ is analytic.
4. Find the image of the region $x + y > 1$ under the mapping $w = 1/z$.
5. Evaluate $\oint_C \frac{e^z}{1+e^z} dz$, $C : |z| = 1$.
6. Find the Taylor series expansion of $f(z) = e^z \sin z$ about $z = 0$.
7. Show that the function $f(z) = \operatorname{cosec} z$ has a simple pole at $z = 0$.
8. Let V be a vector space of all 2x3 matrices over the real field R and W consists of all Matrices for which $A^2 = A$. Show that W is not a subspace of V.
9. Let $A = \begin{bmatrix} -6 & 12 \\ -3 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$. Determine if B is in Col A. Is B in null A ?
10. Find the maximum and minimum values of $Q(x) = 5x_1^2 + 8x_2^2$, if $X = (x_1, x_2)$ and $X^T X = 1$.

PART – B **(20×3=60 Marks)**

Module – 1

11. a) Show that the function $f(z) = \frac{x^3(1+i) - y^3(1-i)}{x^2+y^2}, z \neq 0$
$= 0, z = 0.$
satisfies the CR equations at z = 0 but $f'(0)$ does not exist.
b) Determine the analytic function $f(z) = u + iv$ where $u + v = (x - y)(x^2 + 4xy + y^2)$
c) Discuss the transformation $W = z^2$.

P.T.O.

12. a) Find the analytic function f(z) = u + iv given $u - v = \dfrac{\sin 2x}{\cosh 2y - \cos 2x}$

   b) Show that an analytic function with constant modulus is constant.

   c) Find the bilinear transformation which maps – 1, 0, 1 of the z – plane on to – 1, – i, 1 of the w-plane. Find the fixed point of this transformation.

Module – 2

13. a) Evaluate $\int_C (z - z^2)dz$ where C is the upper half of $|z| = 1$.

   b) Evaluate $\int_0^{2\pi} \dfrac{d\theta}{5 + 4\sin\theta}$

   c) Use residue theorem to evaluate $\int_C \dfrac{dz}{z^3(z+4)} dz$ where C is

   1) $|z| = 2$ 2) $|z + 2| = 3$.

14. a) Evaluate the integral $I = \oint_C \dfrac{e^z}{z^2(z+1)^3} dz$ where c is $|z| = 2$

   b) Expand $\dfrac{1}{z^2 - 3z + 2}$ in the region

   1) $1<|z|<2$ 2) $0<|z-1|<1$

   c) Use contour integration method to evaluate $\int_0^{\infty} \dfrac{dx}{1+x^4}$.

Module – 3

15. a) Solve the equations 2x + 3y + z = 9, x + 2y + 3z = 6, 3x + y + 2z = 8 using LU decomposition.

   b) Find a basis and the dimension of the subspace w of $\mathbb{R}^4$ generated by (1, 4, – 1, 3), (2, 1, – 3, – 1) and (0, 2, 1, – 5).

   c) Find a spanning set for the null space of the matrix $\begin{bmatrix} 1 & 2 & 3 & -1 \\ 3 & 5 & 8 & -2 \\ 1 & 1 & 2 & 0 \end{bmatrix}$.

16. a) Find the singular value decomposition of $\begin{bmatrix} 1 & 3 & 3 \\ -1 & 1 & 2 \end{bmatrix}$.

   b) Find the least square solution of the equation

   $AX = b$ where $A = \begin{bmatrix} -1 & 3 & 3 \\ 1 & 5 & 1 \\ 1 & 7 & -2 \end{bmatrix}$, $b = \begin{bmatrix} -5 \\ 3 \\ 5 \end{bmatrix}$

   c) Test the definiteness of the quadratic form $Q(x) = 2x^2 + 2y^2 - 2z^2 + 2xy + 2yz - 2xz$.

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Time : 3 Hours Max. Marks : 100

Instruction : *Answer **all** questions from Part **A** and **one full** question from **each** Module of Part **B**.*

PART – A **(4×10=40 Marks)**

1. If $f(x) = \begin{cases} K(2x+3), & 0 < x < 2 \\ 0 & , \text{otherwise} \end{cases}$ is a probability density function, find K and F(x).

2. Six coins are tossed 3200 times, using Poisson distribution to obtain approximate probability of getting 6 heads x times.

3. Find the mean and variance of uniform distribution.

4. The mileage which a Car owner gets with a certain kind of tyre is a random variable having an exponential distribution with mean 40,000 kms. Find the probability that one of these tyres will last atleast 30,000 kms.

5. Two lines of regression are $5x - 6y + 90 = 0$ and $15x - 8y = 130$, find $\bar{x}$, $\bar{y}$ and γ.

6. A random sample of 900 items with mean 3.5 is drawn from a population with SD 2.61. Find the 95% confidence interval for the mean.

7. Define (i) Type I error (ii) Type II error (iii) Power of a test and (iv) Critical region.

P.T.O.