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Reg. No. : ....

Name : ....

Fifth Semester B.Tech. Degree Examination, May 2012
(2008 Scheme)
08.501 : ENGINEERING MATHEMATICS – IV (ERFBH)

Time : 3 Hours Max. Marks : 100

Instruction : *Answer **all** questions from Part **A** and **one full** question from **each** Module of Part **B**.*

PART – A **(4×10=40 Marks)**

1. If $f(x) = \begin{cases} K(2x+3), & 0 < x < 2 \\ 0 & , \text{otherwise} \end{cases}$ is a probability density function, find K and F(x).

2. Six coins are tossed 3200 times, using Poisson distribution to obtain approximate probability of getting 6 heads x times.

3. Find the mean and variance of uniform distribution.

4. The mileage which a Car owner gets with a certain kind of tyre is a random variable having an exponential distribution with mean 40,000 kms. Find the probability that one of these tyres will last atleast 30,000 kms.

5. Two lines of regression are $5x - 6y + 90 = 0$ and $15x - 8y = 130$, find $\bar{x}$, $\bar{y}$ and γ.

6. A random sample of 900 items with mean 3.5 is drawn from a population with SD 2.61. Find the 95% confidence interval for the mean.

7. Define (i) Type I error (ii) Type II error (iii) Power of a test and (iv) Critical region.

P.T.O.

8. Suppose X(t) is a process with $\mu(t) = 3$ and $R(t_1, t_2) = 9 + 4e^{-|t_1 - t_2|/5}$. Find variance and covariance of X(5) and X(8).

9. Find the average power of the random process if its spectral density in $S(w) = \frac{1}{1+w^2}$.

10. Define (i) Markov process (ii) Markov chain (iii) Poisson process and (iv) Transition probability matrix.

PART – B

Answer **one full** question from **each** Module. **(3×20=60 Marks)**

Module – I

11. a) A machine manufacturing screws is known to produce 5% defectives. In a random sample of 15 screws, what is the probability that there are (i) exactly three defectives (ii) not more than 3 defectives.

b) In a normal distribution, 5% of the items are under 60 and 40% are between 60 and 65. Find the mean and SD of the distribution.

c) If X has a uniform distribution in (– k, k), k > 0, find k such that $P[|X| < 1] = P[|X| > 1]$.

OR

12. a) Fit a Poisson distribution to the following data :

x :	0	1	2	3	4
f :	63	28	6	2	1

b) In a test on 3000 electric bulbs, it was found that the life of a particular make was normally distributed with an average of 3040 and SD of 60 hours. Estimate the number of bulbs likely to burn for

i) more than 3250 hours

ii) less than 2850 hours and

iii) more than 2920 hours but less than 3160 hours.

c) The time in hours required to repair a machine is exponentially distributed with mean 30 hours. What is the probability that the required time

i) exceeds 35 hours

ii) in between 25 hours and 34 hours

iii) atmost 20 hours.

Module – II

13. a) Use the principle of least squares to fit a straight line to the following data :

x :	0	5	10	15	20
y :	7	11	16	20	26

b) In a sample of 20 persons from a town it was seen that 4 are suffering from T.B. Find a 90% confidence limits for the proportion of T.B. patients in the town.

c) Random samples of sizes 500 and 600 are found to have means 11.5 and 10.5 respectively. Can the samples be regarded as samples drawn from the same population whose SD is 6.

OR

14. a) Find the coefficient of correlation from the following data :

x :	20	22	23	25	25	28	29	30	30	34
y :	18	20	22	24	21	26	26	25	27	29

b) A sample of 10 items gave a mean 4.2 and a SD 2.78. Find a 99% confidence limits for the population mean.

c) Random samples of sizes 500 and 600 are found to have means 11.5 and 10.5 respectively. Can the samples be regarded as samples drawn from the same population whose SD is 6.

Module – III

15. a) The joint pdf of X and Y is given by $f(x,y)=\begin{cases}x+y, & 0<x,y<1\\ 0, & \text{elsewhere}\end{cases}$

Find : (i) $P[X \le \frac{1}{2}]$ (ii) $P[X+Y \le 1]$ (iii) the marginal distributions of X and Y.

b) Show that the random process $X(t) = A \sin(\omega t + \theta)$ where A and ω are constants, θ is uniformly distributed in $(0, 2\pi)$, is WSS.

c) The power spectral density of a random process $\{X(t)\}$ is given by

$S(\omega)=\begin{cases}1+\omega^2, & \text{for } |\omega|\le 1\\ 0, & \text{elsewhere}\end{cases}$. Find its auto correlation function.

OR

16. a) The joint density of X and Y is given by

$f(x,y)=\begin{cases}xy^2+\frac{1}{8}x^2, & 0\le x\le 2, 0<y<1\\ 0, & \text{elsewhere}\end{cases}$. Compute (i) $P(X>1)$ (ii) $P[X+Y\le 1]$ (iii) $P[X<Y]$.

b) Find the spectral density of the random process whose autocorrelation function is $R(\tau)=\frac{1}{2}e^{-|\tau|}$.

c) The tpm of a Markov chain $\{X_n\}$ having 4 states 0, 1, 2, 3 is

$$P=\begin{bmatrix}0 & 1 & 0 & 0\\ 0.2 & 0 & 0.8 & 0\\ 0 & 0.2 & 0 & 0.8\\ 0 & 0 & 1 & 0\end{bmatrix}$$ and the initial distribution is $P[X_0 = i] = 0.25$ for $i = 0, 1, 2, 3$.

Find :

i) $P[X_2=2, X_1=1 \mid X_0=2]$

ii) $P[X_2=2, X_1=1, X_0=2]$

iii) $P[X_2=3]$