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Reg. No. : ....

Name : ....

First Semester M.Tech. Degree Examination, March 2014
(2013 Scheme)
Electronics and Communication
Streams : Telecommunication Engineering, Signal Processing, Microwave and TV Engineering, Communication Systems
TSC 1001 : RANDOM PROCESSES AND APPLICATIONS

Time : 3 Hours Max. Marks : 60

Instruction : *Answer* ***any two*** *questions from* ***each*** *Module.* ***Each*** *question carries* ***10*** *marks.*

Module – I

1. a) It is known that all items produced by a certain machine will be defective with probability 0.1, independently of each other. What is the probability that in a sample of 3 items, at most one will be defective ? 5

   b) A random variable X has exponential distribution with parameter $\lambda = \frac{1}{2}$. Find a transformation Y = g (X) such that the random variable Y is uniform in (2, 4). 5

2. a) If X and Y are two independent random variables and both are uniform in [0, 2], find and plot the probability density function of the random variable $Z = |X-Y|$. 5

   b) If the random variable X_1 is uniformly distributed over (0,4) and X_2 is uniformly distributed over (0, 1), find $P(\max(X_1, X_2) > 3)$. 5

3. Two components of a microcomputer have the following joint pdf for their useful life times X and Y.

$$f_{XY}(x, y) = \begin{cases} xe^{-x}(1+y); & x \geq 0, y \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

   a) What is the probability that the life time X for the first component exceeds 3 ? 5

   b) What is the probability that the life time of at least one component exceeds 3 ? 5

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Module – II

4. a) If X and Y are two independent Poisson random variables with parameters μ_1 and μ_2 respectively, using moment generating functions, compute the probability mass function of $Z = X + Y$. 5

   b) What do you mean by the independent increment property of the Poisson process ? Using this property obtain an expression for the autocorrelation of a Poisson process. 5

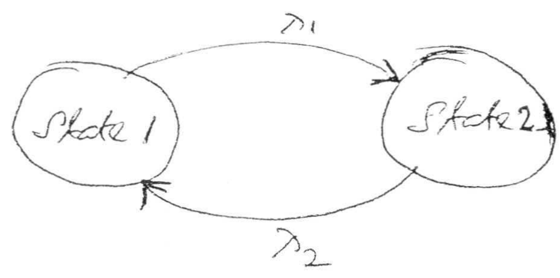
5. Two jointly normal random variables X_1 and X_2 have the joint pdf given by

$$f_{X_1X_2}(x_1, x_2) = \frac{2}{x\sqrt{7}} e - \frac{8}{7}\left(x_1^2 + \frac{3}{2}x_1x_2 + x_2^2\right).$$

   a) Obtain the Co-variance matrix of the random vector $\begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$. 4

   b) Find a transformation A in $\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = A\begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$, such that Y_1 and Y_2 are independent. 6

6. a) Obtain expressions for the steady state probabilities for the two-state birth-death Markov chain shown below. 5



   b) A WSS process X(t) with autocorrelation function $R_X(u)$ is passed through a linear time invatiant system with impulse response h(t). Derive expressions for the autocorrelation and power spectral density of the output process. Y (t). 5

Module – III

7. a) State central Limit Theorem. Using this theorem find the approximate pdf of $Y = \sum_{i=1}^{100} Xi$, if Xi' s are iid with uniform in [1,3]. (2+3)

   b) A WSS random sequence X(n) has the correlation function $R_{xx}(m) = 10e^{-\lambda(m)}$. Find the power spectral density $S_{xx}(\omega)$ for $|\omega| \leq \pi$. Given that $\lambda > 0$. 5

8. a) Using Chebyshev's inequality show that convergence of a random sequence in the mean-square sense implies convergence in probability. 3

   b) If a WSS process X(t) is mean square periodic with period T, show that the KL expansion of X(t) is given by $X(t) = \sum_{n=-\infty}^{\infty} A_n e^{j\omega ot}$ in the mean square sense. Assume that the coefficients An are statistically orthogonal. 7

9. a) A random process X(t) is given by $X(t) = A\cos(\omega_0 t + \varphi)$ where A and ω_0 are constants and φ is a random variable uniform in $[0, 2\pi]$. Show that the process is ergodic in mean and correlation. 7

   b) Check whether the above process satisfies the condition $\int_{-\infty}^{\infty} |R_{xx}(z)|\, dz < \infty$, where $R_{xx}(z)$ is the ensemble autocorrelation of X(t). 3
