First Semester M.Tech. Degree Examination, March 2014  
(2013 Scheme)  
Electronics and Communication  
Stream : Signal Processing  
TSM 1001 – LINEAR ALGEBRA FOR SIGNAL PROCESSING

Time: 3 Hours  Max. Marks : 60

Instructions: Answer any two questions from each Module. 10 marks for each question.

MODULE – 1

1. a) What are the axioms defining a vector space? Give an example for a finite and infinite dimensional vector space.

b) Define linear independence of vectors. If u, v, w ∈ V, vector space are independent show that if u + v, u − v, u − 2v + w are also independent.

2. Write short notes on:
   a) Hilbert space
   b) Lp space
   c) Cyclic subspace
   d) Invariant subspace.

3. a) Define inner product space. If V be a vector space of polynomial with inner product given by < f, g > = ∫₀⁹ f(t) g(t) dt find < f, g > and ‖ f ‖.

b) If W be a vector space of 2 × 2 symmetric matrices over R, what is the dimension of W. Also find a basis of W.  

(10×2=20 Marks)

P.T.O.
MODULE – 2

4. a) Show that the linear operator \( T \in \mathbb{A}(V) \) is invertible only if the constant term in the characteristic polynomial of \( T \) is zero.

b) A linear operator \( T \) is defined by \( T(x, y, z) = (x - 3y + 3z, 3x - 5y + 3z, 6x - 6y + 4z) \). Express \( T \) in diagonal matrix form by suitably selecting a basis.

5. a) Show that similarity transformation does not change the Eigen values of a transformation matrix.

b) State and prove any one property of a projection matrix. Find the projection of \( b \) on to the column space of \( A \)

\[
\begin{bmatrix}
1 & -6 \\
1 & -2 \\
1 & 1 \\
1 & 7
\end{bmatrix} \begin{bmatrix}
-1 \\
2 \\
1 \\
6
\end{bmatrix}
\]

(10x2=20 Marks)

6. a) Show that Null space of linear operator \( T \) is a vector space.

b) Find the inverse of \( B \) using row reduction.

\[
B = \begin{bmatrix}
1 & 0 & 2 \\
2 & -1 & 3 \\
4 & 1 & 8
\end{bmatrix}
\]

MODULE – 3

7. Define unitary transformation. DFT of a sequence \( x(n) \) of length \( N \) is given by

\[
X(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi nk/N}.
\]

Find the DFT transformation matrix for \( N = 4 \). Also show that the columns of a DFT matrix are orthogonal to each other.
8. a) If $\lambda$ is an Eigen value of the invertible operator $T$ show that $\lambda^{-1}$ is an Eigen value of $T^{-1}$.

b) What is Eigen vector of a linear operator, $T$? Write the diagonal form of the matrix $A$ if possible.

$$
A = \begin{bmatrix}
4 & 1 & 4 & -4 \\
0 & 1 & 3 & 8 \\
0 & 0 & 3 & -2 \\
0 & 0 & 0 & 2
\end{bmatrix}
$$

9. a) Write short notes on Jordan canonical form.

b) Find the Jordan canonical form of the matrix

$$
A = \begin{bmatrix}
2 & 0 & 1 & -3 \\
0 & 2 & 10 & 4 \\
0 & 0 & 2 & 0 \\
0 & 0 & 0 & 3
\end{bmatrix}
$$

(10×2=20 Marks)