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Reg. No. : ....

Name : ....

First Semester M.Tech. Degree Examination, March 2014
(2013 Scheme)
Electronics and Communication
Stream : Signal Processing
TSM 1001 – LINEAR ALGEBRA FOR SIGNAL PROCESSING

Time: 3 Hours Max. Marks : 60

***Instructions :** Answer **any two** questions from **each** Module. **10** marks for **each** question.*

MODULE – 1

1. a) What are the axioms defining a vector space ? Give an example for a finite and infinite dimensional vector space.

   b) Define linear independence of vectors. If u, v, w $\in$ V, vector space are independent show that if u + v, u – v, u – 2v + w are also independent.

2. Write short notes on :

   a) Hilbert space

   b) Lp space

   c) Cyclic subspace

   d) Invariant subspace.

3. a) Define inner product space. If V be a vector space of polynomial with inner product given by $<f, g> = \int_0^1 f(t)\,g(t)\,dt$ find $<f, g>$ and $\|f\|$.

   b) If W be a vector space of 2 × 2 symmetric matrices over R, what is the dimension of W. Also find a basis of W. **(10×2=20 Marks)**

P.T.O.

MODULE – 2

4. a) Show that the linear operator $T \in A(V)$ is invertible only if the constant term in the characteristic polynomial of T is zero.

   b) A linear operator T is defined by $T(x, y, z) = (x - 3y + 3z, 3x - 5y + 3z, 6x - 6y + 4z)$. Express T in diagonal matrix form by suitably selecting a basis.

5. a) Show that similarity transformation doses not change the Eigen values of a transformation matrix.

   b) State and prove any one property of a projection matrix. Find the projection of b on to the column space of A

$$v_1 = \begin{bmatrix} 1 & -6 \\ 1 & -2 \\ 1 & 1 \\ 1 & 7 \end{bmatrix} \quad b = \begin{bmatrix} -1 \\ 2 \\ 1 \\ 6 \end{bmatrix}$$

(10×2=20 Marks)

6. a) Show that Null space of linear operator T is a vector space.

   b) Find the inverse of B using row reduction.

$$B = \begin{bmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ 4 & 1 & 8 \end{bmatrix}$$

MODULE – 3

7. Define unitary transformation. DFT of a sequence x (n) of length N is given by $X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-\frac{j2\pi nk}{N}}$. Find the DFT transformation matrix for N = 4. Also show that the columns of a DFT matrix are orthogonal to each other.

8. a) If λ is an Eigen value of the invertible operator T show that λ^{-1} is an Eigen value of T^{-1}.

b) What is Eigen vector of a linear operator, T ? Write the diagonal form of the matrix A if possible.

$$A = \begin{bmatrix} 4 & 1 & 4 & -4 \\ 0 & 1 & 3 & 8 \\ 0 & 0 & 3 & -2 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

9. a) Write short notes on Jordan canonical form.

b) Find the Jordan canonical form of the matrix

$$A = \begin{bmatrix} 2 & 0 & 1 & -3 \\ 0 & 2 & 10 & 4 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}.$$

(10×2=20 Marks)