



Reg. No. :

Name :

First Semester M.Tech. Degree Examination, March 2014
(2013 Scheme)
Electronics and Communication – Telecommunication Engineering
TTM1001 : LINEAR ALGEBRA

Time : 3 Hours

Max. Marks : 60

Instruction : Answer two full questions from each Module.

Module – I

1. a) Let $T : \mathbb{R}^3 \rightarrow \mathbb{R}^2$ be the linear transformation given by $T(a+b,c) = (a+b+c, b+c)$
find the matrix A of T with respect to the standard basis for \mathbb{R}^3 and \mathbb{R}^2 .
- b) Show that the set of all integral multiples of 3 is a ring under ordinary addition and multiplication.
2. State and prove Rank-Nullity theorem.
3. a) Let V be a vector space which is spanned by a finite set of vectors $\beta_1, \beta_2, \dots, \beta_m$. Then, prove that any independent set of vectors in V is finite and contains no more than m elements.
- b) Show that a linear transformation T from V into W , where V and W are vector spaces over the field F , is non singular if and only if T carries each linearly independent subset of V onto a linearly independent subset of W .

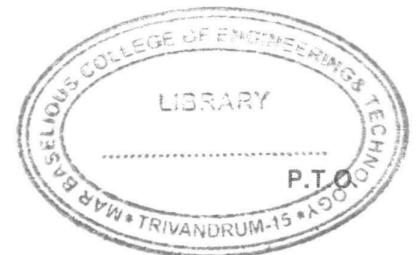
Module – II

4. a) Prove that an orthogonal set of non-zero vectors is linearly independent.
- b) Determine the existence and uniqueness of the solutions to the system.

$$3x_2 - 6x_3 + 6x_4 + 4x_5 = -5$$

$$3x_1 - 7x_2 + 8x_3 - 5x_4 + 8x_5 = 9$$

$$3x_1 - 9x_2 + 12x_3 - 9x_4 + 6x_5 = 15$$





5. a) Verify that the following defines an inner product in \mathbb{R}^2

$$\langle u, v \rangle = x_1 y_1 - x_1 y_2 - x_2 y_1 + 3x_2 y_2 \text{ where } u = (x_1, x_2) \text{ and } v = (y_1, y_2)$$

b) Prove $|\langle x, y \rangle| \leq \|x\| \|y\|$

and $\|x + y\| \leq \|x\| + \|y\|$

6. a) Prove that projection mapping on a Hilbert space is self adjoint and idempotent.
b) Prove that $C[a, b]$ is not a Hilbert space.

Module – III

7. a) Prove that eigen values of a unitary matrix are of magnitude unity.

b) Diagonalize $A = \begin{bmatrix} 1 & -3 & 3 \\ 0 & -5 & 6 \\ 0 & -3 & 4 \end{bmatrix}$.

8. a) Find the QR factorization $A = QR$ of $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

- b) Explain pseudo inverse of a matrix A.

9. a) Find singular value decomposition of $\begin{bmatrix} 6 & 2 \\ -7 & 6 \end{bmatrix}$.

- b) What is the SVD for A^T and A^{-1} ?