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Reg. No. : ....

Name : ....

First Semester M.Tech. Degree Examination, March 2014
Machine Design
MDM 1001 : ENGINEERING MATHEMATICS

Time : 3 Hours Max. Marks : 60

Instruction : *Answer **any two** questions from **each** Module.*

Module – I

1. a) Show that $\vec{V} = 3x^2y^2z^4\hat{i} + 2x^3yz^4\hat{j} + 4x^3y^2z^3\hat{k}$ is irrotational and find its scalar potential.

   b) Verify Stoke's theorem when $\vec{V} = z\hat{i} + (2x + z)\hat{j} + x\hat{k}$ where S is the boundary of the triangle with vertices at (1, 0, 0), (0, 2, 0), (0, 0, 3) oriented upwards.

2. a) Write using summation convention $(x^1)^2 + (x^2)^2 + (x^3)^2 + ... + (x^n)^2$.

   b) Show that $\frac{\partial x^p}{\partial \bar{x}^q}\frac{\partial \bar{x}^q}{\partial x^r} = \delta^p_r$.

   c) A covariant tensor has components $2x - z$, x^2y, yz in Cartesian co-ordinate system. Find its components in spherical co-ordinates.

3. a) Define Skew-symmetric tensor.

   b) Show that a symmetric tensor of second order has only $\frac{1}{2}n(n+1)$ different non-zero components.

Module – II

4. a) Solve the integral equation $y(x) = x + 2\int_0^x y(t)\cos(x - t)\,dt$.

   b) Using method of successive approximation, solve $y(x) = 1 + \lambda\int_0^1 xty(t)\,dt$.

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5. a) Solve the equation $\frac{dy}{dx} = 3\int_0^x \cos 2(x-t)\, y(t)\, dt + 2$, given y (0) = 1.

   b) Classify and reduce the equation $U_{xx} + U_{xy} - 2U_{yy} = 0$ to a canonical form and hence solve it.

6. a) Evaluate the Green's function for the equation $y'' + xy = 1$ given that y (0) = 0 and $y'(1) = 0$.

   b) Solve the Laplace equation subject to the conditions u(0, y) = u (l, y) = u (x, 0) = 0 and u (x, a) = sin n π x/l.

Module – III

7. Solve the Laplace equation $u_{xx} + u_{yy} = 0$ using Liebmann's method satisfying the following conditions : u (0, y) = 0, u (3, y) = 8 + 2y for $0 \le y \le 3$; u (x, 0) = x^2, u (x, 3) = $3x^2$ for $0 \le x \le 3$.

8. Solve $u_t = u_{xx}$ given that u (x, 0) = sin πx, $0 \le x \le 1$ and u (0, t) = u(1, t) = 0 using Schmidt method. (Take h = 0.2 and α = 1/2).

9. a) Prove that $\frac{d}{dx}\left[x^n J_n(x)\right] = x^n J_{n-1}(x)$.

   b) Prove that $(1 - 2xt + t^2)^{\frac{-1}{2}} = \sum_{n=0}^{\infty} t^n P_n(x)$.