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Reg. No. : ....

Name : ....

First Semester M.Tech. Degree Examination, March 2014
(2013 Scheme)
MECHANICAL ENGINEERING (Machine Design)
MDC 1003 : Continuum Mechanics

Time : 3 Hours Total Marks : 60

Instruction : *Answer* ***any 2*** *questions from* ***each*** *Module.*

MODULE – I

1. a) Prove that $\in_{pqs}\in_{snr} = \delta_{pn}\delta_{qr} - \delta_{pr}\delta_{qn}$. 5

   b) If a second order tensor A is given by $A_{ij}e_i \otimes e_j$ and v a first order tensor given by $v_p e_p$ show that m^{th} component of A acting on v i.e. $A_{mp}v_p$ is given by $\left[\left(A_{ij}\, \underset{\sim}{e}_i \otimes \underset{\sim}{e}_j\right)\left(v_p\, \underset{\sim}{e}_p\right)\right] \cdot e_m$. 5

2. a) Derive the tensorial transformation law $\tilde{T} = Q^T T Q$ using indicial notation. 5

   b) The tensor T has components $\begin{bmatrix} 1 & 0 & 3 \\ 0 & 2 & 2 \\ 3 & 2 & 4 \end{bmatrix}$.

   Find the components of the tensor in a new reference frame found through a rotation of 60° counter-clockwise about the x_3 axis. Show that the invariants will not change by the transformation. 5

3. A deformation of the body is given by $x_1 = 4X_1$, $x_2 = -\frac{1}{2}X_2$, $x_3 = -\frac{1}{2}X_3$. If the Cauchy stress tensor for this body is

   $$\begin{bmatrix} 100 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{MPa}$$

   Find out :

   a) First Piola-Kirchhoff stress tensor

   b) Second Piola-Kirchhoff stress tensor. 10

MODULE – II

4. A continuum body undergoes the deformation $x_1 = X_1$, $x_2 = X_2 + AX_3$, $x_3 = X_3 + AX_2$ where A is a constant.

   Compute the Lagrangian and Almansi stress tensors. For the above deformation, show that the stretch λ is unity for line elements parallel to the X_1 axis. **10**

5. a) The spatial velocity gradient tensor is given by

$$L = \frac{\partial v_i}{\partial x_j} e_i \otimes e_j$$

   Show that it can be decomposed into the vorticity tensor [W] and rate of deformation tensor [D] which are anti-symmetric and symmetric respectively. **5**

   b) Given the velocity gradient $v_1 = 16x_2$ $v_2 = v_3 = 0$. Find out the rate of deformation tensor and vorticity tensor. **5**

6. From the principle of mass balance of a continuum derive the Lagrangian and Eulerian forms of continuity equation. **10**

MODULE – III

7. Starting from the equation for balance of linear momentum

$$\text{div}\ \underset{\sim}{\sigma} + \rho \underset{\sim}{b} = \rho \frac{\partial \underset{\sim}{v}}{\partial x}$$

   show that for plane stress conditions $\nabla^2(\sigma_{11} + \sigma_{22}) = 0$ making use of compatibility conditions. **10**

8. Write down the constitutive matrix of a linear elastic solid possessing three mutually perpendicular planes of elastic symmetry. Name the material. How do you reduce the matrix to that of an isotropic material by successive rotation about the three axes of elastic symmetry using transformation matrices ? Also express the same in terms of Lame's constants. **10**

9. A large thin plate containing a small circular hole of radius 'a' is subjected to simple uniaxial tension. Determine the field of stress. Use the stress function

$$\phi = \left(Ar^2 + Br^4 + C/r^2 + D\right)\cos 2\theta \cdot$$ **10**

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