Reg. No. : ....

Name : ....

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First Semester M.Tech. Degree Examination, March 2014
(2013 Scheme)
Branch : ELECTRICAL AND ELECTRONICS ENGINEERING
Streams : Control Systems, Power Control and Drives, Guidance and Navigational Control
EMA 1002 : Applied Mathematics

Time : 3 Hours | Max. Marks : 60

Instructions : *Answer **any two** questions from **each** Module.*
***All** Modules are **compulsory**.*
***Each** question carries **10** marks.*

MODULE – I

1. a) Let W be the subspace of R^4 spanned by $x_1 = (1, 2, 1, -2)$, $x_2 = (2, 3, 2, -3)$ and $x_3 = (2, 5, 2, -5)$. Find a basis for W and the dimension of W.

   b) Give the basis $s = \{x_1 = (2, 4, -4), x_2 = (-3, 6, 0), x_3 = (7, 2, 1)\}$ of R^3. Use Gram-Schmidt process to find an orthonormal basis.

2. a) Find a linear transformation $T : R^3 \longrightarrow R^4$ whose range is generated by (1, 0, –1, 2) and (1, 2, 2, –1).

   b) Prove that a linear transformation $T : V \longrightarrow W$ is one-to-one if Ker T = { 0 }.

   c) If x_1, x_2, x_3 are linearly independent vectors in a vector space V, show that the vectors, $x_2 + x_3, x_3 + x_1, x_1 + x_2$ are also linearly independent.

3. Find a Singular value decomposition of $\begin{bmatrix} 1 & -1 \\ -2 & 2 \\ 2 & -2 \end{bmatrix}$.

MODULE – II

4. a) Find the curves on which the functional $\int_0^1 \left((y')^2 + 12xy\right)dx$ with y(0) = 0 and y(1) = 1 can be extremised.

   b) Solve $y(x) = x + 2\int_0^x \cos(x - t)\, y(t)dt$ using convolution.

P.T.O.

5. a) Find a function y(x) for which $\int_0^1 (x^2 + y'^2)\,dx$ is stationary, given that $\int_0^1 y^2\,dx = 2$, $y(0) = 0$, $y(1) = 0$.

   b) By method of successive approximation solve $y(x) = 1 + \lambda \int_0^1 (1 - 3xt)\,y(t)\,dt$.

6. Solve $\frac{\partial u}{\partial t} = 2\frac{\partial^2 u}{\partial x^2}$ subject to the conditions $u(0, t) = 0$, $u(5, t) = 0$, $u(x, 0) = \sin \pi x$.

MODULE – III

7. a) If $\{X(t)\}$ is a Gaussian process with $\mu(t) = 10$ and $C(t_1, t_2) = 16\, e^{-|t_1 - t_2|}$ find the probability that (i) $X(10) \le 8$ and (ii) $|X(10) - X(6)| \le 4$.

   b) Classify different states of Markov Chain.

8. a) Three boys A, B and C are throwing a ball to each other. 'A' always throws the ball to C, but C is just as likely to throw the ball to B as to A. Show that the process is Markovian. Find the transition probability matrix and classify the states.

   b) At what average rate must a clerk in a supermarket work in order to ensure a probability of 0.90 that the customer will not wait longer than 12 minutes ? It is assumed that there is only one counter at which customers arrive in Poisson fashion at an average rate of 15 per hour and that the length of the service by the Clerk has an exponential distribution.

9. a) Draw the state diagram of a birth-death process and obtain the balance equation. Hence find the limiting distribution of the process.

   b) Explain M|M|1 model with infinite capacity. Also write the Little's formula.