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Reg. No. : ....

Name : ....

First Semester M.Tech. Degree Examination, March 2014
(2013 Scheme)
Electronics and Communication – Telecommunication Engineering
TTM1001 : LINEAR ALGEBRA

Time : 3 Hours Max. Marks : 60

***Instruction** : Answer **two** full questions from **each** Module.*

Module – I

1. a) Let $T : \mathbb{R}^3 \to \mathbb{R}^2$ be the linear transformation given by T (a+b,c) = (a+b+c,b+c) find the matrix A of T with respect to the standard basis for $\mathbb{R}^3$ and $\mathbb{R}^2$.

   b) Show that the set of all integral multiples of 3 is a ring under ordinary addition and multiplication.

2. State and prove Rank-Nullity theorem.

3. a) Let V be a vector space which is spanned by a finite set of vectors $\beta_1, \beta_2, \ldots, \beta_m$. Then, prove that any independent set of vectors in V is finite and contains no more than m elements.

   b) Show that a linear transformation T from V into W, where V and W are vector spaces over the field F, is non singular if and only if T carries each linearly independent subset of V onto a linearly independent subset of W.

Module – II

4. a) Prove that an orthogonal set of non-zero vectors is linearly independent.

   b) Determine the existence and uniqueness of the solutions to the system.

$$3x_2 - 6x_3 + 6x_4 + 4x_5 = -5$$
$$3x_1 - 7x_2 + 8x_3 - 5x_4 + 8x_5 = 9$$
$$3x_1 - 9x_2 + 12x_3 - 9x_4 + 6x_5 = 15$$

P.T.O.

5. a) Verify that the following defines an inner product in $\mathbb{R}^2$

$$\langle u, v\rangle = x_1 y_1 - x_1 y_2 - x_2 y_1 + 3x_2 y_2 \text{ where } u = (x_1, x_2) \text{ and } v = (y_1, y_2)$$

b) Prove $|\langle x, y\rangle| \le \|x\| \, \|y\|$

and $\|x + y\| \le \|x\| + \|y\|$

6. a) Prove that projection mapping on a Hilbert space is self adjoint and idempotent.

b) Prove that c[a, b] is not a Hilbert space.

Module – III

7. a) Prove that eigen values of a unitary matrix are of magnitude unity.

b) Diagonalize $A = \begin{bmatrix} 1 & -3 & 3 \\ 0 & -5 & 6 \\ 0 & -3 & 4 \end{bmatrix}$.

8. a) Find the QR factorization A = QR of $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

b) Explain pseudo inverse of a matrix A.

9. a) Find singular value decomposition of $\begin{bmatrix} 6 & 2 \\ -7 & 6 \end{bmatrix}$.

b) What is the SVD for A^T and A^{-1} ?
