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Reg. No. : ....

Name : ....

First Semester M.Tech. Degree Examination, March 2013
(2008 Scheme)
(Electronics & Communication Engineering)
Stream : Telecommunication Engineering
TTC1004 : PROBABILITY AND RANDOM PROCESS FOR COMMUNICATION

Time : 3 Hours Max. Marks : 100

Instructions : *1) Answer* ***any five*** *questions.*
2) ***All*** *questions carry* ***equal*** *marks.*

1. a) Show that if $S = \int_0^{10} X(t)dt$, then $E[s^2] = \int_{-10}^{10} (10-|\tau|)A_x(\tau)d\tau$, where $Ax(\tau)$ is the autocorrelation function of the process X(t).

   b) Find the mean and variance of S if $E[X(t)] = 8$ and $R_X(\tau) = 64 + 10\, e^{-2|\tau|}$.

2. a) Show that the Poisson process has independent increments.

   b) Obtain an expression for the autocorrelation of the Poisson process.

   c) Show that if a Gaussian process is wide sense stationary, it is strict sense stationary also.

3. a) White noise is passed through an RC lowpass filter with cut-off frequency 1 KHz. Find the mean square value and autocorrelation of the output process.

   b) A source transmits a signal θ with probability density function given by

   $$f_\theta(\theta) = \frac{1}{2\pi}, 0 \le \theta \le 2\pi$$
   $$= 0, \text{ otherwise}$$

   Because of additive Gaussian noise, the probability density function of the received signal Y when $\theta = \theta$ is

   $$f(y/\theta) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(y-\theta)^2}{2\sigma^2}}$$

   Compute the mean value of the received signal.

P.T.O.

4. a) State and prove the central limit theorem.

b) A random process X(t) is given by $X(t) = A\cos(\omega_0 t + \varphi)$, where A and ω_0 are constants and φ is a random variable uniform in $[0, 2\pi]$. Check whether the process is ergodic in mean and autocorrelation.

5. a) A signal x(t) is given by

$$x(t) = A,\ 0 \le t \le T/2$$

$$= -A,\ T/2 < t \le T$$

$$= 0, \text{ elsewhere}$$

Draw a signal y(t) which is orthogonal to x(t) and show that they are orthogonal. The signal y(t) should be such that y(t) = 0, t < 0 and t > T.

b) A zero mean normal random vector $X = (X_1, X_2)^T$ has covariance matrix C given by $C = \begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix}$.

Calculate the mean and variance of $Y = A^T X + B$, where $A = (2, -1)^T$ and $B = 5$.

6. a) A random sequence X(n) has the autocorrelation given by $R_x[n] = a^{|n|}, -\infty < n < \infty$. Find the power spectral density of the process.

b) What is a whitening filter ? Obtain the transfer function of a whitening filter for the above process.