First Semester M.Tech. Degree Examination, March 2013
(2008 Scheme)
(Electronics & Communication Engineering)
Stream : Telecommunication Engineering
TTC1004 : PROBABILITY AND RANDOM PROCESS FOR COMMUNICATION

Time : 3 Hours

Max. Marks : 100

Instructions: 1) Answer any five questions.
              2) All questions carry equal marks.

1. a) Show that if \( S = \int_0^{10} X(t)dt \), then \( E[s^2] = \int_{-10}^{10} (10 - |\tau|) A_x(\tau)d\tau \), where \( A_x(\tau) \) is
       the autocorrelation function of the process \( X(t) \).

   b) Find the mean and variance of \( S \) if \( E[X(t)] = 8 \) and \( R_x(\tau) = 64 + 10 \ e^{-2|\tau|} \).

2. a) Show that the Poisson process has independent increments.

   b) Obtain an expression for the autocorrelation of the Poisson process.

   c) Show that if a Gaussian process is wide sense stationary, it is strict sense
      stationary also.

3. a) White noise is passed through an RC lowpass filter with cut-off frequency
      1 KHz. Find the mean square value and autocorrelation of the output process.

   b) A source transmits a signal \( \theta \) with probability density function given by

      \[ f_\theta(\theta) = \begin{cases} \frac{1}{2\pi} , & 0 \leq \theta \leq 2\pi \\ 0 , & \text{otherwise} \end{cases} \]

      Because of additive Gaussian noise, the probability density function of the
      received signal \( Y \) when \( \theta = \theta \) is

      \[ f(y / \theta) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(y-\theta)^2}{2\sigma^2}} \]

      Compute the mean value of the received signal.
4. a) State and prove the central limit theorem.
   
b) A random process \( X(t) \) is given by \( X(t) = A \cos(\omega_c t + \phi) \), where \( A \) and \( \omega_c \) are constants and \( \phi \) is a random variable uniform in \([0, 2\pi]\). Check whether the process is ergodic in mean and autocorrelation.

5. a) A signal \( x(t) \) is given by

\[
x(t) = \begin{cases} 
  A, & 0 \leq t \leq \frac{T}{2} \\
  -A, & \frac{T}{2} < t \leq T \\
  0, & \text{elsewhere}
\end{cases}
\]

Draw a signal \( y(t) \) which is orthogonal to \( x(t) \) and show that they are orthogonal. The signal \( y(t) \) should be such that \( y(t) = 0 \), \( t < 0 \) and \( t > T \).

b) A zero mean normal random vector \( X = (X_1, X_2)^T \) has covariance matrix \( C \) given by

\[
C = \begin{bmatrix} 
  3 & -1 \\
  -1 & 3 
\end{bmatrix}
\]

Calculate the mean and variance of \( Y = A^TX + B \), where \( A = (2, -1)^T \) and \( B = 5 \).

6. a) A random sequence \( X(n) \) has the autocorrelation given by

\[
R_X[n] = a^n, -\infty < n < \infty .
\]
Find the power spectral density of the process.

b) What is a whitening filter? Obtain the transfer function of a whitening filter for the above process.