Combined First and Second Semester B.Tech. Degree
Examination, May 2011
(2008 Scheme)
08-101 : ENGINEERING MATHEMATICS – I
(CMNPETARUF)
9. Verify Cayley-Hamilton theorem for \( A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix} \) and hence find \( A^{-1} \).

10. Show that \( 8x_1^2 + 7x_2^2 + 3x_3^2 - 12x_1x_2 - 8x_2x_3 + 4x_1x_3 \) is positive semidefinite.

PART - B

Answer two questions from each Module. Each question carries 10 marks.

Module - I

11. a) If \( y = \frac{\sin \frac{-1}{x}}{\sqrt{1+x^2}} \)

   Show that \( (1+x^2) \frac{y_{n+2}}{y_{n+1}} + (2n+3)x \frac{y_{n+1}}{y_n} + (n+1)^2 \frac{y_{n-1}}{y_n} = 0 \).

   b) The focal length \( 'f' \) of a mirror is given by \( \frac{1}{v} - \frac{1}{u} = \frac{1}{f} \). Show that the relative error in \( f \) is \( k \left( \frac{1}{u} + \frac{1}{v} \right) \) where \( \Delta u = \Delta v = k \).

12. Evaluate \( \int_0^\infty \frac{dx}{a^2 + x^2} \) (\( a > 0 \)) and hence deduce the values of \( \int_0^\infty \frac{dx}{(a^2 + x^2)^2} \) and \( \int_0^\infty \frac{dx}{(a^2 + x^2)^3} \).

13. a) Find the directional derivative of \( \phi = 4xz^3 - 3x^2y^2z \) at \( (2, -1, 2) \) in the direction of \( 2i - 3j + 6k \).

   b) If \( \mathbf{F} = (x^2 - yz)i + (y^2 - xz)j + (z^2 - xy)k \), show that \( \mathbf{F} \) is irrotational and find its scalar potential.
14. a) Find \( L^{-1} \left[ \tan^{-1} \left( \frac{f}{s} \right) \right] \).

b) Find the Laplace transform of the waveform \( f(t) = \frac{2t}{3}, 0 \leq t \leq 3 \) of period 3.

15. Solve the differential equation \((D^2 - 5D + 6) y = x^2 + e^x \cos 2x\).

16. Solve the simultaneous differential equation:

\[
\frac{dx}{dt} - x = y; \quad \frac{dy}{dt} = 4x - 2y.
\]

Module – III

17. a) Find the rank of the Matrix

\[
\begin{bmatrix}
2 & 3 & -1 & -1 \\
1 & -1 & -2 & -4 \\
3 & 1 & 3 & -2 \\
6 & 3 & 0 & -7
\end{bmatrix}
\]

b) Show that the vectors \((1, 0, 2), (0, 1, 2), (0, 0, 2)\) are linearly independent.

18. Test for consistency and solve

\[
4x - 2y + 6z = 8 \\
x + y - 3z = -1 \\
15x - 3y + 9z = 21
\]

19. Diagonalize the Matrix

\[
\begin{bmatrix}
-1 & 2 & -2 \\
1 & 2 & 1 \\
-1 & -1 & 0
\end{bmatrix}
\]